Math 2433 **Exam 2** **Name**______________

April 9, 2010 PS #______________

NO CALCULATORS!

1. Determine which of the following vector fields is the gradient of a function $f(x,y)$. If it is, find all such functions $f(x,y)$. 18 pts

   a. $\mathbf{V}(x,y) = \left(2xe^{y} - \sin(x)\right)\mathbf{i} + \left(x^2e^{y} + \cos(2y)\right)\mathbf{j}$

   b. $\mathbf{W}(x,y) = \left(x^2 - e^{2y}\right)\mathbf{i} + \left(2xy - e^{2x}\right)\mathbf{j}$

2. If $\nabla f(3,2) = 3\mathbf{i} - 4\mathbf{j}$, $\mathbf{r}(t) = 3(4t+1)\mathbf{i} + 2e^{4t}\mathbf{j}$ and $h(t) = f(\mathbf{r}(t))$: find $h'(0)$. 9 pts

3. Suppose $f(x,y) = x\cos(y) + y\cos(z) + z\sin(x)$. 12 pts
   Find $f_{xx} + f_{yy} + f_{zz}$.

4. Find an equation for the tangent plane and scalar parametric equations for the normal line to the surface $xy^2 + 2z^2 = 20$, at the point (x,y,z) = (2,1,3). 8 pts

   a. Tangent plane:

   b. Scalar parametric equations for normal line: 6 pts

5. a. Find a unit vector in the direction in which the function $f(x,y,z) = x^2ze^{y} + xz^2$ increases most rapidly, at $(x,y,z) = (1,\ln 2,\ 2)$. 8 pts

   b. What is the directional derivative of f at $(1,\ \ln 2,\ 2)$ in this direction? 6 pts

   c. Find the directional derivative of f in the direction of $2\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}$ at $(x,y,z) = (1,\ln 2,\ 2)$. 6 pts

6. Find the maximum and minimum values of: $f(x,y) = x^2 - 6x + 2y^2 - 16y + 21$ on the domain $D = \{(x,y):\ 2 \le x \le 5,\ 1 \le y \le 6\}$, and find all points on this domain at which these values occur. 13 pts

7. Find all of the critical points of the function

$$f(x,y) = \frac{y^3}{3} + 2xy + x^2 - 3y$$

   and determine whether each critical point yields a local maximum value, local minimum value, or saddle point. 14 pts