Department of Mathematics, University of Houston
Topology - Blecher
Qualifying Exam

Instructions: Put all your bags and papers on the side of the room. R^n is Euclidean n-space, $R = R^1$. Answer as many questions as you can.

1. (a) Which subspaces of a compact Hausdorff space are compact?
   (b) What is a locally compact space?
   (c) Prove that an open subspace of a compact Hausdorff space is locally compact.
   (d) Is every locally compact Hausdorff space homeomorphic to an open subspace of a compact Hausdorff space? Say why or give a counterexample.

2. (a) What is a first countable topological space?
   (b) What is a net, and what does it mean for a net in a topological space to converge?
   (c) Is a convergent net in R bounded? Prove it or give a counterexample.
   (d) Prove that in a compact first countable space, every sequence has a convergent subsequence.

3. (a) How is a quotient topology defined?
   (b) Show that the quotient topology obtained from R by identifying two numbers if they differ by a rational number, is the indiscrete topology.

4. State as many characterizations as you know of separable metric spaces.

5. (a) How is the product topology defined?
   (b) Show that the 'projection map' from a product topological space $\prod_{j \in J} X_j$ (with the product topology) to one of the spaces X_j, is an open map.
   (c) Let X be the product of an infinite countable number of copies of the two point set $\{0, 1\}$ with its usual (discrete) topology. Give X the product topology. What topological properties does it have? Is it normal? Metrizable? Compact? Explain. What are its connected components?

6. (a) Define the fundamental group $\pi_1(X, x_0)$, its group operations, and say a few words about why it is a group.
   (b) Show that R^2 is not homeomorphic to R^3.

7. (a) What is a manifold?

(b) Show that any manifold is metrizable. Is it paracompact? Normal? Explain.

8. (a) What is a covering map?

   (b) If $\pi : E \to B$ is a covering map between path connected spaces, and if Y is simply connected, show that π is a homeomorphism.

9. (a) Prove that the n-sphere S^n is simply connected for $n > 1$.

   (b) What is a simple closed curve in a topological space X?

   (c) State the Jordan curve theorem in R^2.

   (d) State the Jordan separation theorem in R^2, and say some words about its proof.