Section 5.4 - Inverse Trigonometry

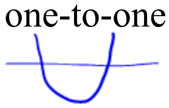
RECALL – Facts about inverse functions:

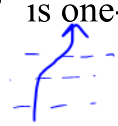
A function $f(x)$ is **one-to-one** if no two different inputs produce the same output (or: passes the horizontal line test)

Example: $f(x)=x^2$ is NOT one-to-one. $g(x)=x^3$ is one-to-one.

$f(-1)=1$ $f(-2)=4$

$f(1)=1$ $f(2)=4$





A function $f(x)$ is **invertible** if it is one-to-one.

The inverse function is notated as: $f^{-1}(x) \leftarrow$ "f inverse"

$f: A \to B$	$f^{-1}: B \to A$
Domain: A	Domain: B
Range: B	Range: A

Important: $f(a)=b$ if and only if $f^{-1}(b)=a$.

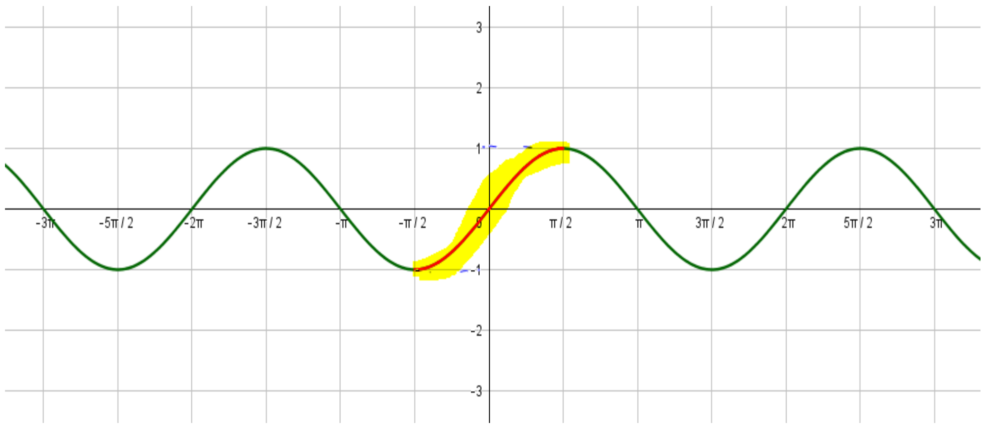
$f(2)=5 \Rightarrow f^{-1}(5)=2$

INVERSE SINE FUNCTION

Here's the graph of $f(x) = \sin(x)$.

Domain: $(-\infty, \infty)$

Range: $[-1, 1]$



Is it one to one? NO

$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

$\sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$

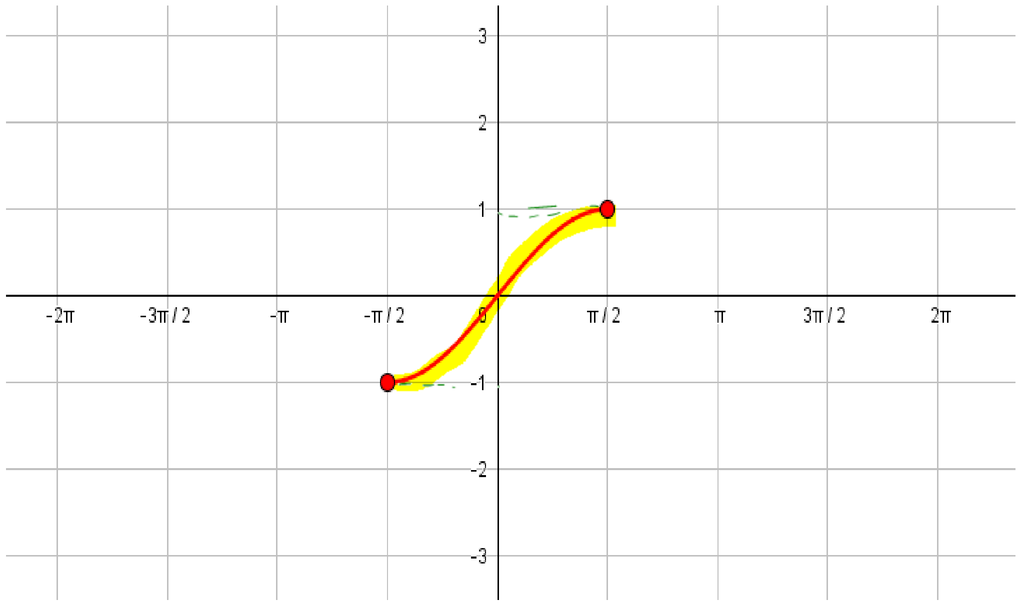
If the function is not one-to-one, we run into problems when we consider the inverse of the function. What we want to do with the sine function is to restrict the values for sine. When we make a careful restriction, we can get something that IS one-to-one.

If we limit the function to the interval $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$, the graph will look like this:

Restricted Sine function

Domain: $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

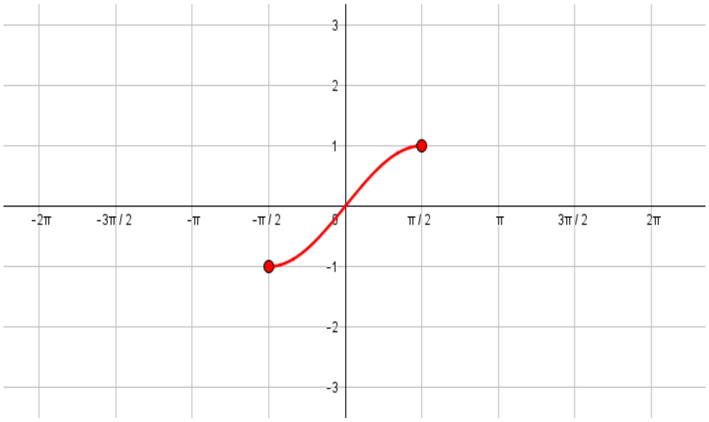
Range: $[-1,1]$



On this limited interval, we have a one-to-one function.

INVERSE SINE FUNCTION

Here's the graph of restricted sine function:



$f^{-1}(x) = \sin^{-1}(x)$

$f: \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to [-1, 1]$

$f^{-1}: [-1, 1] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

numbers

output angles in radian

Restricted Sine function	Inverse Sine Function:
$f(x) = \sin(x)$	$\sin^{-1}(x)$ or $\arcsin(x)$
Domain: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	Domain: $[-1, 1]$
Range: $[-1, 1]$	Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (quadrants 1 and 4)

$f(a) = b \qquad f^{-1}(b) = a$

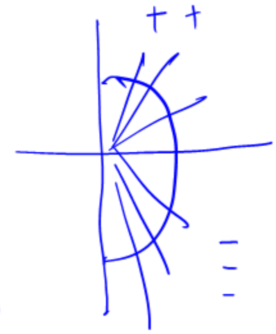
Example: $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2} \to \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

input

output

$\sin^{-1}\left(\frac{1}{2}\right)$

$\sin^{-1}(x)$: $[-1,1] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



Example: $\sin^{-1}\left(\frac{1}{2}\right) = ?$ $\frac{\pi}{6}$

Question: What is the angle in $\underbrace{\left[-\frac{\pi}{2},\frac{\pi}{2}\right]}$ whose sine is 1/2?

range of $\sin^{-1}(x)$

Example: $\sin^{-1}(1) = ?$ angle!

Question: What is the angle in $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ whose sine is 1?

$\sin(\theta) = 1$

$\theta = \frac{\pi}{2}$

$\sin^{-1}(1) = \frac{\pi}{2}$

Example: $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = ?$ $\frac{\pi}{3}$

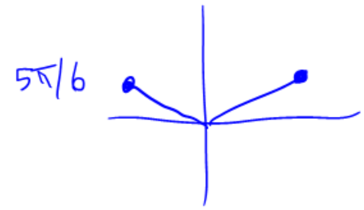
Question: What is the angle in $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ whose sine is $\frac{\sqrt{3}}{2}$?

$\sin(\theta) = \frac{\sqrt{3}}{2}$

$\theta = \frac{\pi}{3}$

must know unit circle!

Important: When we covered the unit circle, we saw that there were two angles that had the same value for most of our angles. With inverse trig functions, this will not happen since we start with restricted functions that are one-to-one. We'll have one quadrant in which the values are positive and one quadrant where the values are negative. The restricted graphs we looked at can help us know where these values lie. We'll only state the values that lie in these intervals (same as the intervals for our graphs):



Example: $\sin\left(\dfrac{\pi}{6}\right)=\dfrac{1}{2}$ and $\sin\left(\dfrac{5\pi}{6}\right)=\dfrac{1}{2}$;

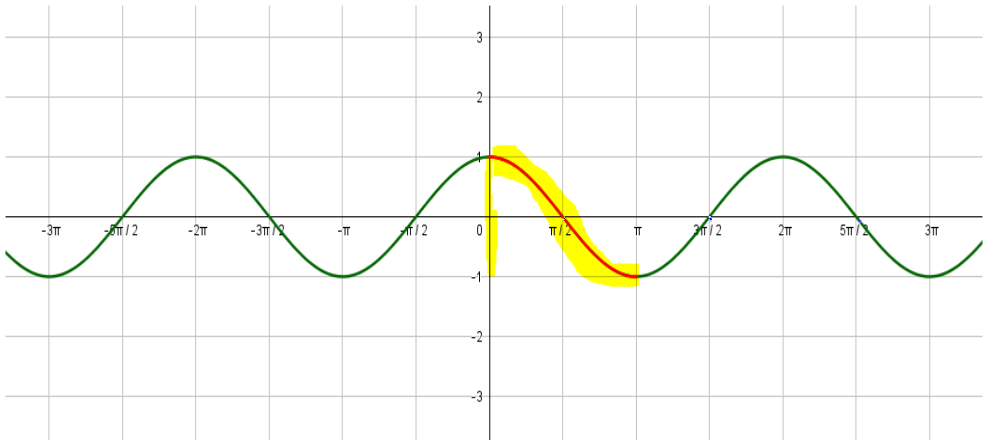
However, $\sin^{-1}\left(\dfrac{1}{2}\right)=\dfrac{\pi}{6}$ (unique answer!) since $\dfrac{5\pi}{6}$ is Not in the range of inverse since function.

INVERSE COSINE FUNCTION
Let's do the same thing with $f(x) = \cos(x)$.

Here's the graph of $f(x) = \cos(x)$.

Domain: $(-\infty, \infty)$

Range: $[-1, 1]$



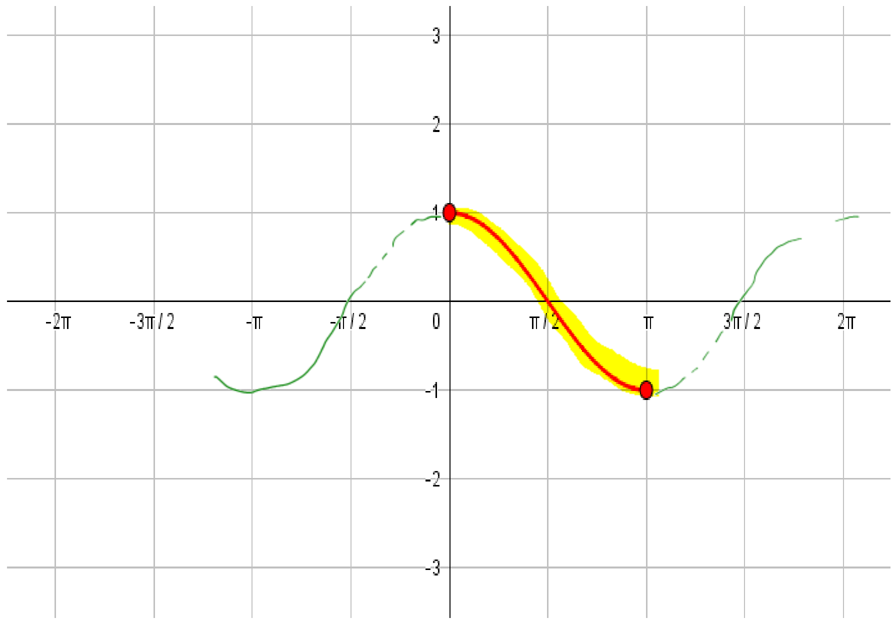
It's not one-to-one. If we limit the function to the interval $[0, \pi]$, however, the function IS one-to-one.

Here's the graph of the restricted cosine function.

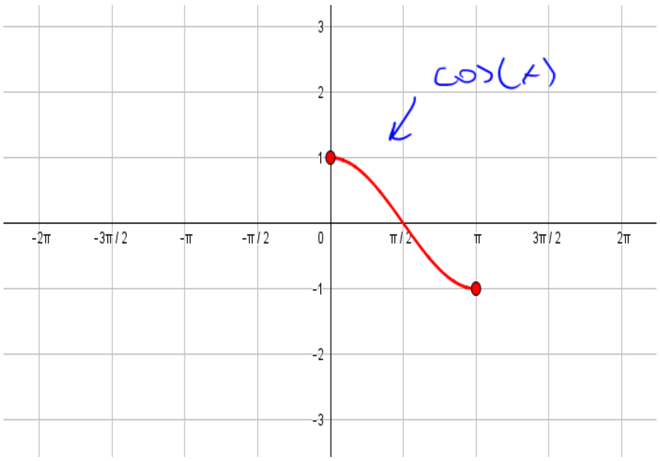
Restricted Cosine function One-to-one

Domain: $\left[0,\pi\right]$

Range: $\left[-1,1\right]$



INVERSE COSINE FUNCTION
Now, let's work on defining the inverse of cosine function.
Here's the graph of restricted cosine function:



$\cos x : [0, \pi] \to [-1, 1]$

$\cos^{-1}(x) : \underset{\text{input}}{[-1, 1]} \to \underbrace{[0, \pi]}_{\text{range}}$

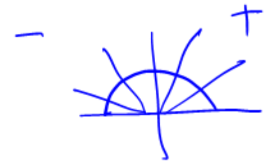
$\cos^{-1}\left(\frac{1}{2}\right) = \text{angle}$

Restricted Cosine function	**Inverse Cosine Function:**
$f(x) = \cos(x)$	$\cos^{-1}(x)$ **or** $\boxed{\arccos(x)}$
Domain: $[0, \pi]$	Domain: $[-1, 1]$
Range: $[-1, 1]$	Range: $[0, \pi]$ (quadrants 1 and 2)

$f(a) = b \qquad f^{-1}(b) = a$

Example: $\cos\left(\dfrac{\pi}{3}\right) = \dfrac{1}{2} \;\to\; \cos^{-1}\left(\dfrac{1}{2}\right) = \dfrac{\pi}{3}$

$\cos^{-1}\left(\frac{1}{2}\right) = ?$



Example: $\cos^{-1}\left(\frac{1}{2}\right) = ?$ $\frac{\pi}{3}$

unit circle

Question: What is the angle in $[0, \pi]$ whose cosine is $\frac{1}{2}$?

range

$\cos(\theta) = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

Example: $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = ?$ $\frac{\pi}{4}$

Question: What is the angle in $[0, \pi]$ whose cosine is $\frac{\sqrt{2}}{2}$?

$\cos(\theta) = \frac{\sqrt{2}}{2}$

unit circle

$\theta = \frac{\pi}{4}$

Example: $\cos^{-1}(1) = ?$ 0

Question: What is the angle in $[0, \pi]$ whose cosine is 1?

$\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$\cos(\theta) = 1$

$\theta = 0$

$\cos(0) = 1$

$\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

$\arccos(1) = 0$

POPPER for Section 5.4:

Question#1: What is the RANGE of $g(x) = \cos^{-1}(x)$ **?**

INVERSE TANGENT FUNCTION

Here's the graph of $f(x) = \tan(x)$. Is it one-to-one?

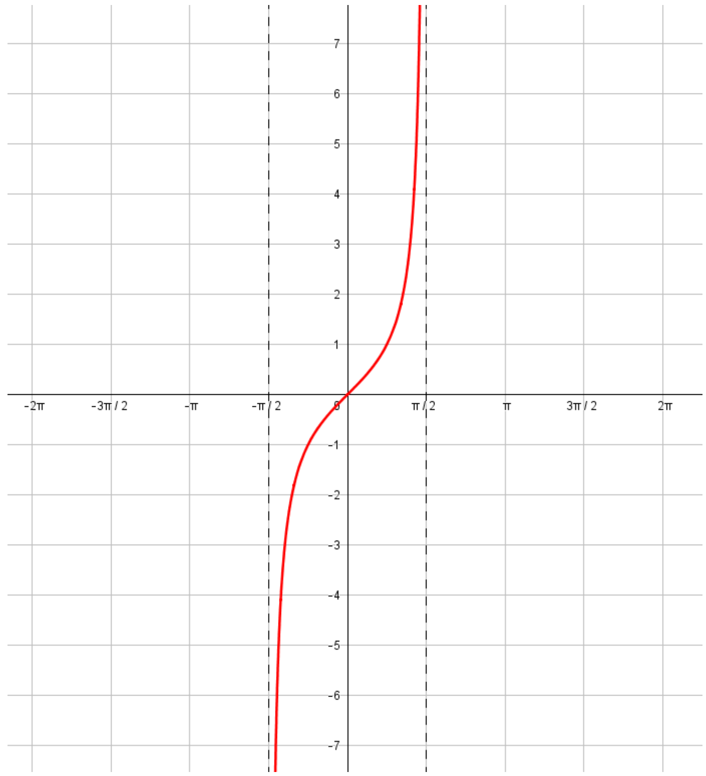
If we restrict the function to the interval $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$, then the restricted function IS one-to-one.

( ): end points excluded

Restricted Tangent function

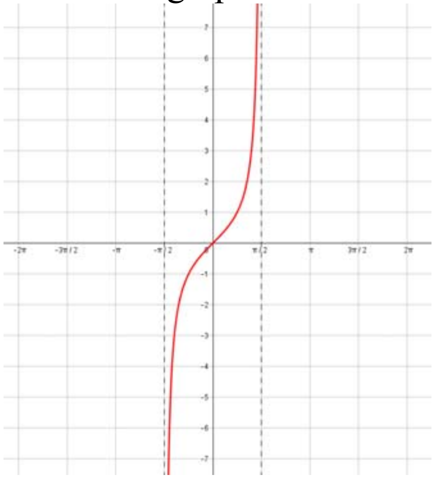
Domain: $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Range: $(-\infty,\infty)$



Inverse Tangent Function

Here's the graph of restricted tangent function:



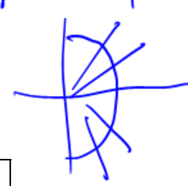
$\tan(x): \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to (-\infty, \infty)$

$\tan^{-1}(x): (-\infty, \infty) \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

numbers angles

$\tan^{-1}(5)$ $\tan^{-1}(1)$

+



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Restricted Tangent function	**Inverse Tangent Function:**
$f(x) = \sin(x)$	$\underline{\tan^{-1}(x)}$ or $\underline{\arctan(x)}$
Domain: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$	Range: $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ (quadrants 1 and 4)

Example: $\tan\left(\frac{\pi}{4}\right) = 1 \quad \to \quad \tan^{-1}(1) = \frac{\pi}{4}$

$\tan^{-1}(1)$

Example: $\tan^{-1}(1) = ?$ $\frac{\pi}{4}$

$\tan(\theta) = 1$

$\theta = \frac{\pi}{4}$

Question: What is the angle in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ whose tangent is 1?

$\uparrow$ range

Example: $\tan^{-1}(0) = ?$ 0

Question: What is the angle in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ whose tangent is 0?

$\tan(\theta) = 0$

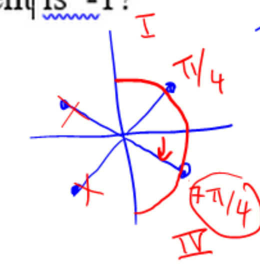
$\theta = 0$

Example: $\tan^{-1}(-1) = ?$ $-\frac{\pi}{4}$

Question: What is the angle in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ whose tangent is -1?

$\tan(\theta) = -1$

$\theta = -\frac{\pi}{4}$



$\leftarrow -\frac{\pi}{4}$

Note: We always give inverse trig angles in **radians**.

Example 1: Compute each of the following:

a) $\cos^{-1}(0) = \frac{\pi}{2}$ $\qquad \cos(\alpha) = 0$

$\alpha = \frac{\pi}{2}$

unit circle!

b) $\tan^{-1}\left(\sqrt{3}\right) = \frac{\pi}{3}$ $\qquad \tan(\alpha) = \sqrt{3}$

$\alpha = \frac{\pi}{3}$

c) $\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$ $\qquad \sin(\alpha) = -\frac{1}{2}$

$\alpha = -\frac{\pi}{6}$

$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

$\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$

d) $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$ $\qquad \sin(\alpha) = -\frac{\sqrt{2}}{2}$

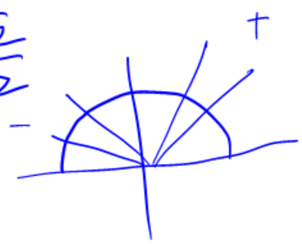
$\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$\sin\left(-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

$\cos^{-1}$

e) $\arccos\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$ $\qquad \cos(\alpha) = -\frac{\sqrt{2}}{2}$

$\alpha = \frac{3\pi}{4}$



Example 2: Compute

$$\arcsin\left(\frac{1}{2}\right)+\arccos(0)+\arctan(-1)$$

$\sin^{-1}\left(\frac{1}{2}\right)+\cos^{-1}(0)+\tan^{-1}(-1)$

$$= \frac{\pi}{6} + \frac{\pi}{2} + -\frac{\pi}{4}$$

(2) (6) (3)

$$= \frac{2\pi + 6\pi - 3\pi}{12}$$

$$= \boxed{\frac{5\pi}{12}}$$

$\arccos(0) = \alpha$

$\cos(\alpha) = 0$

$\alpha = \frac{\pi}{2}$

$\arctan(-1) = \alpha$

$\tan(\alpha) = -1$

$\alpha = -\frac{\pi}{4}$

POPPER for Section 5.4

Question#2: Find the following sum:

$$\arcsin\left(\frac{\sqrt{2}}{2}\right) + \arccos(0)$$

Note: If you need to compute **inverse secant or inverse cosecant** functions:

Question: $\sec^{-1}(2) = ?$

$\sec(\theta) = \frac{1}{\cos(\theta)}$

First, call it an angle: $\sec^{-1}(2) = \theta$

Then, convert: $\sec(\theta) = 2$ flip

Now, express this in terms of cosine: $\cos(\theta) = \frac{1}{2}$

And answer according to "inverse cosine function": $\theta = \frac{\pi}{3}$

Final answer: $\underbrace{\sec^{-1}(2) = \frac{\pi}{3}}$

Question: $\csc^{-1}(1) = ?$

First, call it an angle: $\csc^{-1}(1) = \theta$

Then, convert: $\csc(\theta) = 1$ flip $\frac{1}{1}$

Now, express this in terms of sine: $\sin(\theta) = 1$

And answer according to "inverse sine function": $\theta = \frac{\pi}{2}$

Final answer: $\underbrace{\csc^{-1}(1) = \frac{\pi}{2}}$

NOTE: Domains of inverse trig functions:

$f(x)=\sin^{-1}(x)$; [-1,1] sin⁻¹(5): undef. ~~sin(x)=5~~

$f(x)=\cos^{-1}(x)$; [-1,1]

$f(x)=\tan^{-1}(x)$; $(-\infty,\infty)$

$f(x)=\cot^{-1}(x)$; $(-\infty,\infty)$

$f(x)=\sec^{-1}(x)$; $(-\infty,1]\cup[1,\infty)$ Range of sec(x)

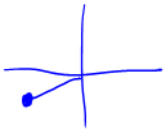
$f(x)=\csc^{-1}(x)$; $(-\infty,1]\cup[1,\infty)$

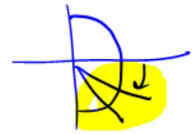
greater than 1

For example; $\sin^{-1}(2)$ or $\cos^{-1}\left(\sqrt{2}\right)$ are not defined.

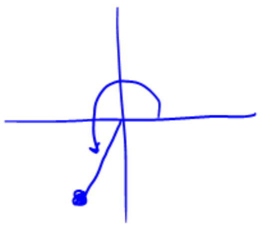
Composition of a trig function with its inverse:

Example 3: Find the exact value: $\sin^{-1}\left[\underbrace{\sin\left(\frac{7\pi}{6}\right)}_{\text{unit circle}}\right]$. $= \sin^{-1}\left(-\frac{1}{2}\right) = \boxed{-\frac{\pi}{6}}$





Example 4: Find the exact value: $\cos^{-1}\left[\cos\left(\frac{4\pi}{3}\right)\right]$. $= \cos^{-1}\left(-\frac{1}{2}\right) = \boxed{\frac{2\pi}{3}}$



$\cos(\alpha) = -\frac{1}{2}$

2nd quad

Example 5: Find the exact value: $\tan^{-1}\left[\tan\left(\frac{3\pi}{4}\right)\right]$. $= \tan^{-1}(-1) = \boxed{-\frac{\pi}{4}}$

Note: If a trigonometric function and its inverse are composed, then we have a shortcut. However, we need to be careful about giving an answer that is in the range of the inverse trig function.

range of $\sin^{-1}(x)$

$$\sin^{-1}(\sin(x)) = x \quad \text{when} \quad x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\cos^{-1}(\cos(x)) = x \quad \text{when} \quad x \in [0, \pi]$$

$$\tan^{-1}(\tan(x)) = x \quad \text{when} \quad x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

Examples: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ ✓ not in the range

$$\cancel{\sin}^{-1}\left[\cancel{\sin}\left(\frac{\pi}{8}\right)\right] = \frac{\pi}{8} \quad \text{but} \quad \sin^{-1}\left[\sin\left(\frac{7\pi}{8}\right)\right] \neq \frac{7\pi}{8}$$

$$\cos^{-1}\left[\cos\left(\frac{\pi}{8}\right)\right] = \frac{\pi}{8} \quad \text{but} \quad \cos^{-1}\left[\cos\left(\frac{9\pi}{8}\right)\right] \neq \frac{9\pi}{8}$$

$$\tan^{-1}\left[\tan\left(\frac{\pi}{8}\right)\right] = \frac{\pi}{8} \quad \text{but} \quad \tan^{-1}\left[\tan\left(\frac{7\pi}{8}\right)\right] \neq \frac{7\pi}{8}$$

If the inverse trig function is the inner function, then our job is easier:

$$\sin(\sin^{-1}(x)) = x \qquad \text{when} \qquad x \in [-1, 1]$$
$$\cos(\cos^{-1}(x)) = x \qquad \text{when} \qquad x \in [-1, 1]$$
$$\tan(\tan^{-1}(x)) = x \qquad \text{when} \qquad x \in (-\infty, \infty)$$

Examples:

$$\sin\left[\sin^{-1}\left(\frac{1}{5}\right)\right] = \frac{1}{5}$$

$$\cancel{\sin(\underbrace{\sin^{-1}(2)}_{\text{undefined}}) = ?}$$

$$\cos\left[\cos^{-1}\left(-\frac{2}{7}\right)\right] = -\frac{2}{7}$$

$$\tan[\tan^{-1}(5)] = 5 \ .$$

POPPER for Section 5.4

Question#3: Evaluate: $\sin\left(\sin^{-1}\left(\frac{4}{5}\right)\right)$

Let's work with composition of different trig and inverse trig functions:

Example 6: Find the exact value: $\cos\Big[\underbrace{\sin^{-1}\left(\frac{5}{13}\right)}_{\alpha}\Big]$. $= \cos(\alpha) = \boxed{\frac{12}{13}}$

$\sin^{-1}\left(\frac{5}{13}\right) = \alpha$

$\sin(\alpha) = \frac{5}{13}$

Given: $\sin(\alpha) = \frac{5}{13}$, find $\cos(\alpha) = ?$

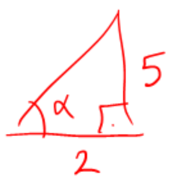
$\cos\alpha = \frac{12}{13}$

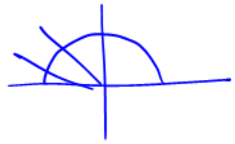
Example 7: Find the exact value: $\tan\left(\underbrace{\cot^{-1}\left(\frac{2}{5}\right)}_{\alpha}\right) = \tan(\alpha) = \boxed{\frac{5}{2}}$

$\cot^{-1}\left(\frac{2}{5}\right) = \alpha$

$\cot(\alpha) = \frac{2}{5}$; $\tan(\alpha) = ?$

$\tan(\alpha) = \dfrac{1}{\cot(\alpha)} = \dfrac{1}{2/5} = \dfrac{5}{2}$





Example 8: Find the exact value: $\tan\left[\underbrace{\cos^{-1}\left(-\frac{4}{5}\right)}_{\alpha\,:\,2^{nd}\text{ quadrant}}\right]$. $= \tan(\alpha) = \boxed{\frac{-3}{4}}$

$\sin : +$

$\cos^{-1}\left(-\frac{4}{5}\right) = \alpha$

$\cos(\alpha) = \frac{-4}{5}$

$\sin^2(\alpha) = 1 - \cos^2(\alpha) = 1 - \left(-\frac{4}{5}\right)^2 = 1 - \frac{16}{25} = \frac{9}{25}$

$\sin(\alpha) = \pm\sqrt{\frac{9}{25}} = +\frac{3}{5}$

"2"

$\tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)} = \frac{3/5}{-4/5} = -\frac{3}{4}$

Example 9: Find the exact value: $\sin\left(\underbrace{\arccos\left(-\frac{1}{4}\right)}_{\alpha\text{ : 2nd quad.}}\right) = \sin(\alpha) = \boxed{\frac{\sqrt{15}}{4}}$

SM: +

$\arccos\left(-\frac{1}{4}\right) = \alpha$

$\cos(\alpha) = -\frac{1}{4}$

$\sin^2(\alpha) = 1 - \cos^2(\alpha) = 1 - \left(\frac{-1}{4}\right)^2 = 1 - \frac{1}{16} = \frac{15}{16}$

$\Rightarrow \quad \sin(\alpha) = \pm\sqrt{\frac{15}{16}} = \frac{\sqrt{15}}{4}$

"2"

Example 10: Find the exact value: $\tan\left(\underbrace{\sec^{-1}(2)}_{\alpha}\right) =$ $\tan(\alpha) = \boxed{\sqrt{3}}$

$\sec^{-1}(2) = \alpha$

$\sec(\alpha) = 2$

$\cos(\alpha) = \frac{1}{2}$ $\left(\text{or } \alpha = \frac{\pi}{3} \Rightarrow \tan\left(\frac{\pi}{3}\right) = \sqrt{3}\right)$

$\tan(\alpha) = \frac{\sqrt{3}}{1} = \sqrt{3}$

Example 11: Find the exact value: $\sin\left(\underbrace{\cos^{-1}(-4)}_{\alpha}\right) =$ undefined

$\cos^{-1}(-4) = \alpha$

$\cos(\alpha) = -4$

Not possible

Example 12: Let $y = \arctan\left(\frac{x}{4}\right)$ where $x > 0$. Express $\boxed{\cos(y)}$ in terms of x.

switch: $\tan(y) = \frac{x}{4}$

angle

$x^2 + 4^2 = c^2$

$x^2 + 16 = c^2$

$c = \sqrt{x^2+16}$

$$\cos(y) = \frac{\text{adj}}{\text{hyp}} = \boxed{\frac{4}{\sqrt{x^2+16}}}$$

POPPER for Section 5.4

Question#4: Evaluate: $\tan\left(\sin^{-1}\left(\frac{3}{5}\right)\right)$

Graphs of Inverse Trigonometric Functions

We note the inverse sine function as $f(x) = \sin^{-1}(x)$ or $f(x) = \arcsin(x)$.

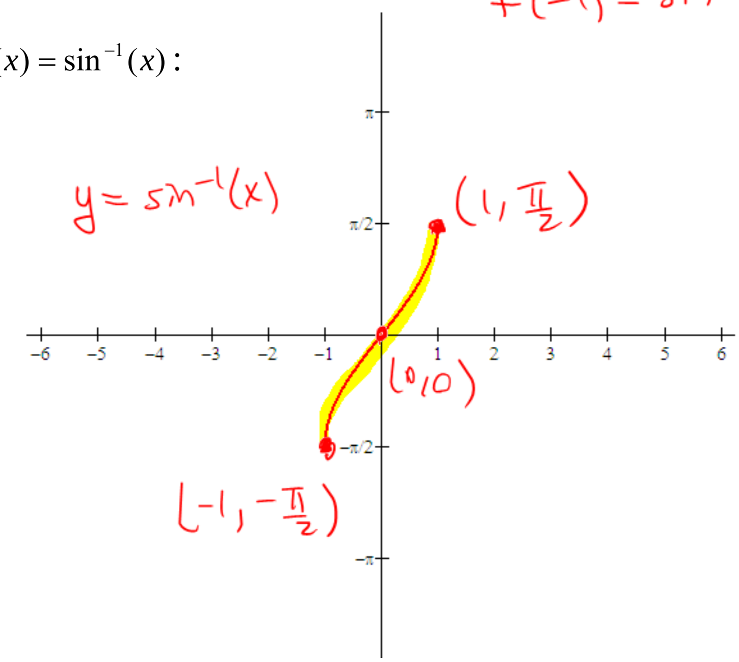
$f(1) = \sin^{-1}(1) = \frac{\pi}{2}$

$f(0) = \sin^{-1}(0) = 0$ $(0,0)$

$f(-1) = \sin^{-1}(-1) = -\frac{\pi}{2}$ $\left(-1, -\frac{\pi}{2}\right)$

Domain: [-1, 1]
Range: $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$.
Key points: $\left(-1, -\frac{\pi}{2}\right), (0,0), \left(1, \frac{\pi}{2}\right)$

Here is the graph of $f(x) = \sin^{-1}(x)$:



Inverse Cosine Function

We note the inverse cosine function as $f(x)=\cos^{-1}(x)$ or $f(x)=\arccos(x)$.

Domain: $[-1,1]$

Range: $[0,\pi]$

$f(-1)=\cos^{-1}(-1)=\pi$

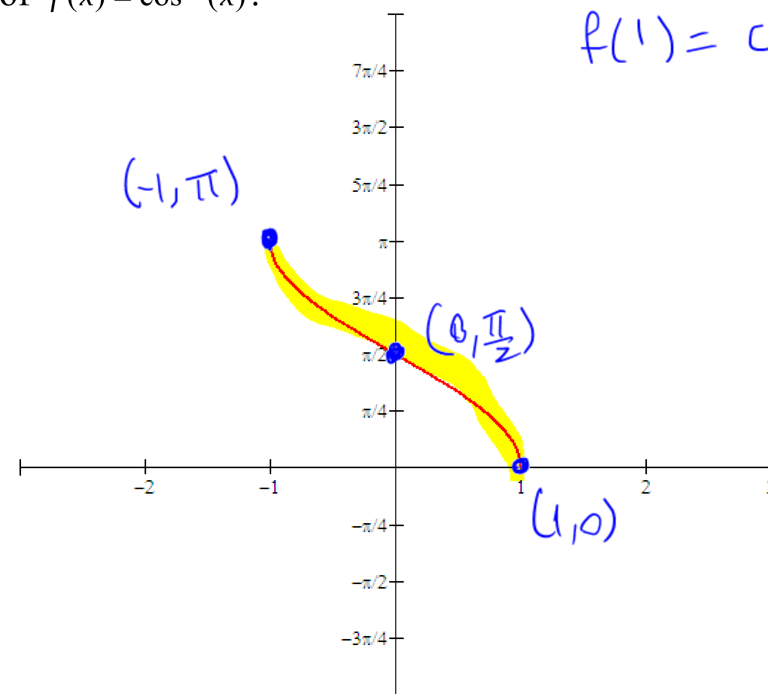
$(-1,\pi)$ ON

$f(0)=\cos^{-1}(0)=\frac{\pi}{2}$

$f(1)=\cos^{-1}(1)=0$

Key points: $(-1,\pi),\left(0,\frac{\pi}{2}\right),(1,0)$

Here is the graph of $f(x)=\cos^{-1}(x)$:

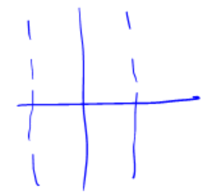


Inverse Tangent Function:

We note the function as $f(x)=\tan^{-1}(x)$ or $f(x)=\arctan(x)$.

Domain: $(-\infty,\infty)$

Range: $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$

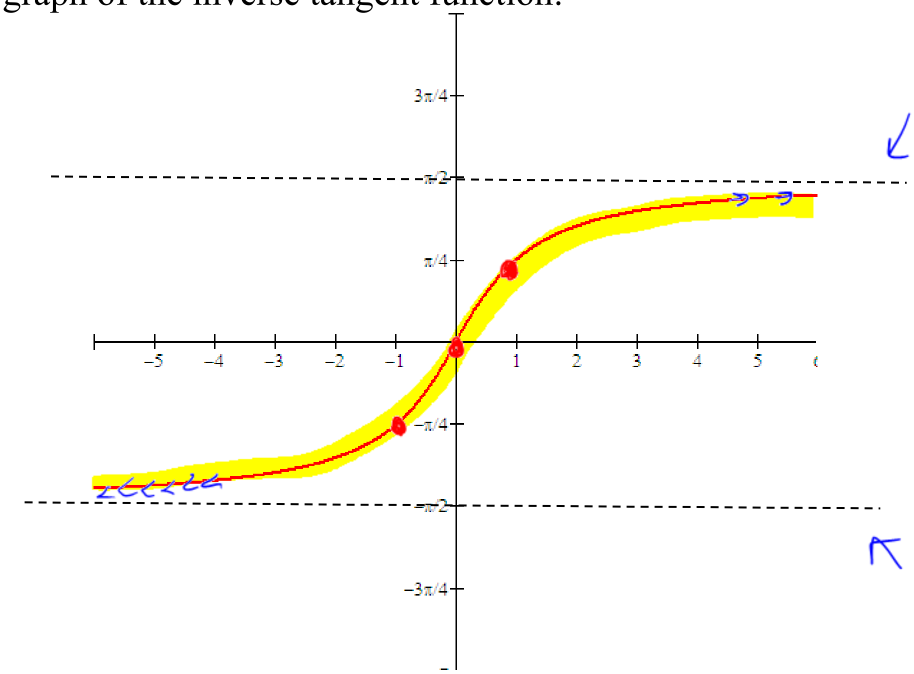


Key points: $\left(-1,-\frac{\pi}{4}\right),(0,0),\left(1,\frac{\pi}{4}\right)$

$(0,0)$ $\tan^{-1}(0)=0$

$(1,\frac{\pi}{4})$ $\tan^{-1}(1)=\frac{\pi}{4}$

Here is the graph of the inverse tangent function:



$y=\pi/2$

$y=-\frac{\pi}{2}$

Important:

Inverse tangent function has two horizontal asymptotes: $y=\frac{\pi}{2}$ and $y=-\frac{\pi}{2}$.

You can use graphing techniques learned in earlier lessons to graph transformations of the basic inverse trig functions.

(a, b) is ON if $f(a)=b$

Example 1: Which of the following points is on the graph of $f(x) = \arctan(x-1)$?

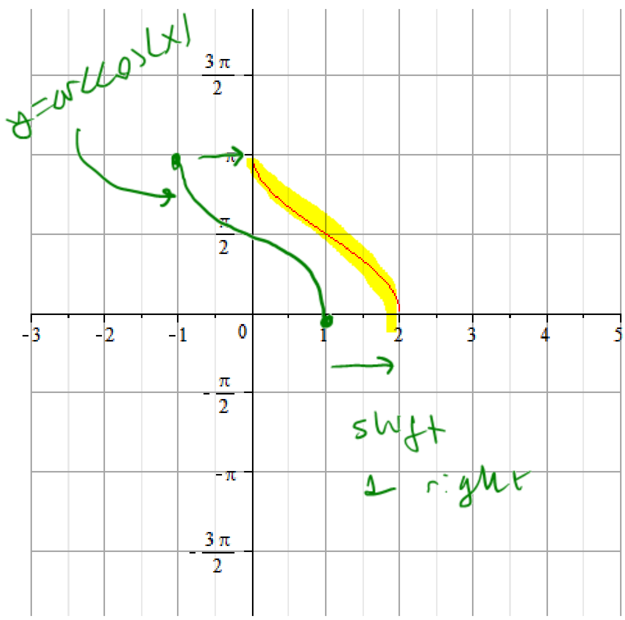
X A) $\left(\frac{\pi}{4}, 0\right)$ $f\left(\frac{\pi}{4}\right) = \arctan\left(\frac{\pi}{4} - 1\right) \overset{?}{=} 0$ X

X B) $\left(0, \frac{\pi}{4}\right)$ X $f(0) = \arctan(0-1) = \arctan(-1) = -\frac{\pi}{4}$

Not $\frac{\pi}{4}$

C) $\left(0, -\frac{\pi}{4}\right)$ $f(0) = -\frac{\pi}{4}$ ✓

Example 2: Which of the following can be the function whose graph is given below?



A) $f(x) = \cos^{-1}(x-1)$

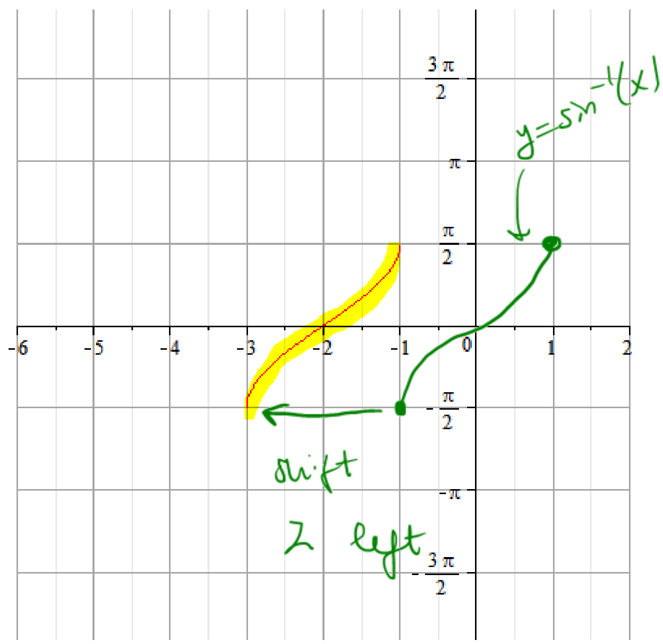
B) $f(x) = \sin^{-1}(x-1)$

C) $f(x) = \cos^{-1}(x+1)$

D) $f(x) = \sin^{-1}(x+1)$

E) $f(x) = \tan^{-1}(x-1)$

Example 3: Which of the following can be the function whose graph is given below?



A) $f(x) = \cos^{-1}(x-2)$

B) $f(x) = \sin^{-1}(x-2)$

C) $f(x) = \cos^{-1}(x+2)$

D) $f(x) = \sin^{-1}(x+2)$

E) $f(x) = \tan^{-1}(x+2)$

POPPER for Section 5.4

Question#5: Which of the following points is ON the graph of $f(x) = \arcsin(x+2)$?