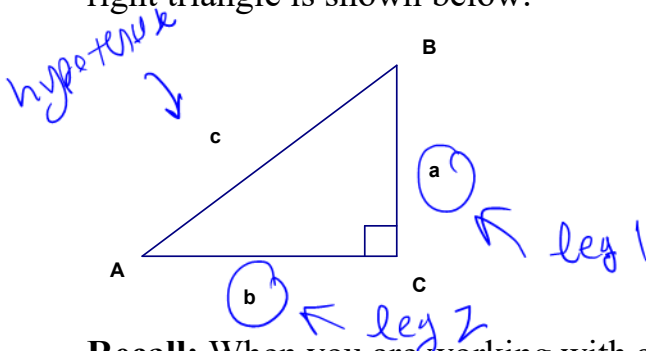


Math 1330 - Section 4.1 Special Triangles

In this section, we'll work with some special triangles before moving on to defining the six trigonometric functions.

Two special triangles are $30^\circ - 60^\circ - 90^\circ$ triangles and $45^\circ - 45^\circ - 90^\circ$ triangles. With little additional information, you should be able to find the lengths of all sides of one of these special triangles.

First we'll review some conventions when working with triangles. We label angles with capital letters and sides with lower case letters. We'll use the same letter to refer to an angle and the side opposite it, although the angle will be a capital letter and the side will be lower case. In this section, we will work with right triangles. Right triangles have one angle which measures 90 degrees, and two other angles whose sum is 90 degrees. The side opposite the right angle is called the hypotenuse, and the other two sides are called the legs of the triangle. A labeled right triangle is shown below.



$$a^2 + b^2 = c^2$$

Recall: When you are working with a right triangle, you can use the **Pythagorean Theorem** to help you find the length of an unknown side. In a right triangle ABC with right angle C,

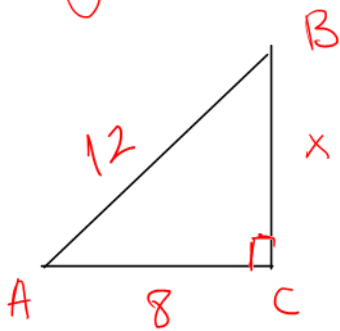
$$a^2 + b^2 = c^2.$$

↓
REFER to "Geometry Module" under:

<https://online.math.uh.edu/courses/placement/Modules.html>

for more help on Pythagorean theorem and areas of triangles.

Example: In right triangle ABC with right angle C, if $AB = 12$ and $AC = 8$, find BC.



$$x^2 + 8^2 = 12^2$$

$$x^2 + 64 = 144$$

$$\begin{array}{r} -64 \end{array} \quad \begin{array}{r} -64 \end{array}$$

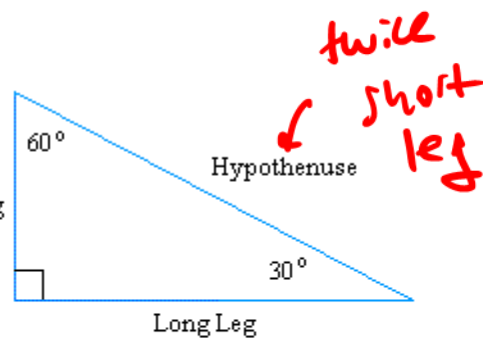
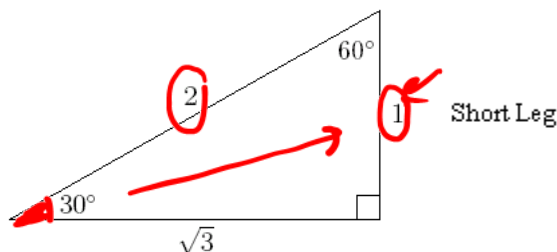
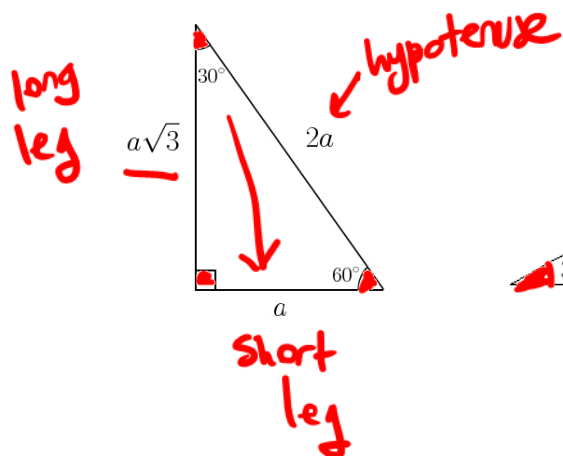
$$x^2 = 80$$

$$\Rightarrow x = \sqrt{80} = \boxed{4\sqrt{5}}$$

↖
16 ÷ 5

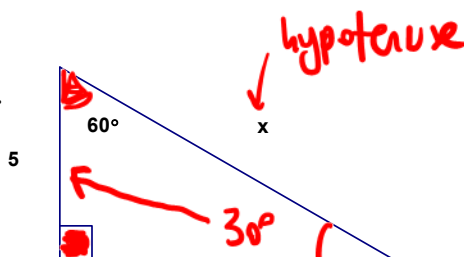
Special Triangles

In a $30^\circ - 60^\circ - 90^\circ$ **triangle**, the lengths of the sides are proportional. If the shorter leg (the side opposite the 30° degree angle) has length a , then the longer leg has length $\sqrt{3}a$ and the hypotenuse has length $2a$. An easy way to remember this is to write the lengths in ratio form as $a : \sqrt{3}a : 2a$. So if you know (or can find) a , you can find the lengths of all three sides.



$$\text{Short Leg} = \frac{1}{2} \text{ Hypotenuse}$$

Example 1: Find x .

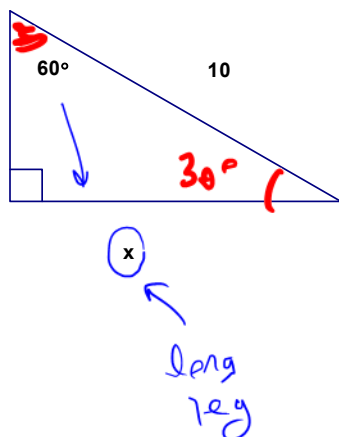


$$\text{short leg} = 5$$

$$\text{hypotenuse} = 2 \cdot 5 = 10$$

$$x = 10$$

Example 2: Find x .



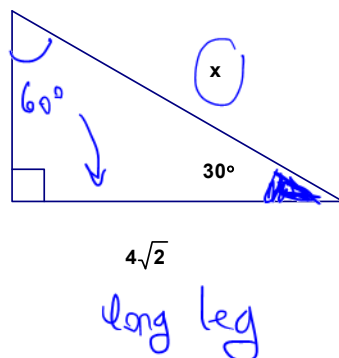
$$\text{hypotenuse} = 10$$

$$\text{short leg} = \frac{\text{hyp}}{2} = \frac{10}{2} = 5$$

$$\text{long leg} = \text{short leg} \cdot \sqrt{3}$$

$$x = 5 \cdot \sqrt{3}$$

Example 3: Find x .



$$\text{long leg} = \text{short} \cdot \sqrt{3}$$

$$\text{short leg} = \frac{\text{long leg}}{\sqrt{3}}$$

$$= \frac{4\sqrt{2}}{\sqrt{3}}$$

$$x = \text{hyp.} = 2 \cdot \text{short leg}$$

$$x = 2 \cdot \frac{4\sqrt{2}}{\sqrt{3}} = \frac{8\sqrt{2} \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}}$$

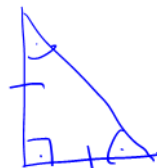
$$x = \frac{8\sqrt{6}}{3}$$

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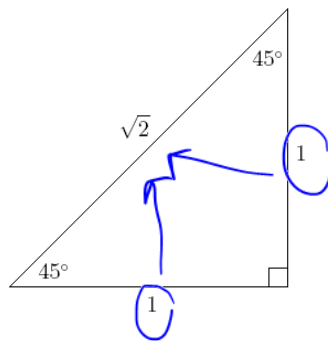
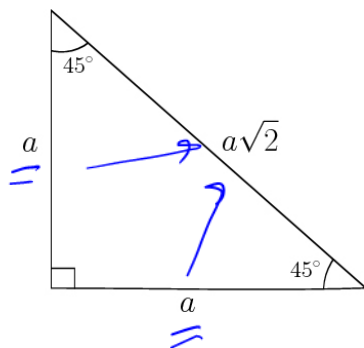
Question 1: In right triangle ABC with right angle C, $m(\angle A) = 30^\circ$ and length of BC is 6. Find the length of the hypotenuse.

Choices can be seen on the video.

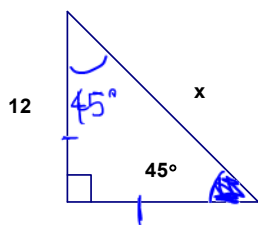
↓ isosceles



In a $45^\circ - 45^\circ - 90^\circ$ triangle, the **two legs of the triangle are equal in length**. This triangle is also called an isosceles right triangle. If the legs each measure a , then the hypotenuse has length $\sqrt{2}a$. The ratio to remember for these triangles is $a : a : \sqrt{2}a$. Again, if you know a , you can find the lengths of all three sides.



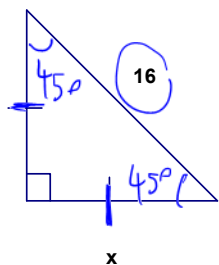
Example 4: Find x .



$$\text{leg: } 12$$

$$x = 12 \cdot \sqrt{2}$$

Example 5: Find x .



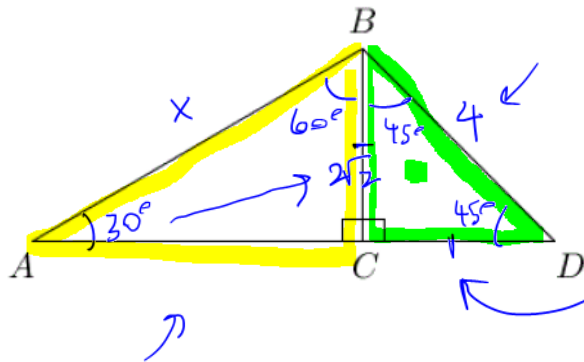
$$\text{hyp} = 16 = \text{leg} \cdot \sqrt{2}$$

$$\text{leg} = \frac{\text{hyp}}{\sqrt{2}} = \frac{16 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{16\sqrt{2}}{2} = \boxed{8\sqrt{2}}$$

Nice!

Example 6: Right triangles ACB and BCD are given below. If $|BC| = |CD|$, $m(\angle A) = 30^\circ$ and $|BD| = 4$, find the length of AB.

isosceles
right



hyp. : 4

$$\text{leg} = \frac{4 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$$

each!

$$BC = 2\sqrt{2}$$

$$30^\circ - 60^\circ - 90^\circ$$

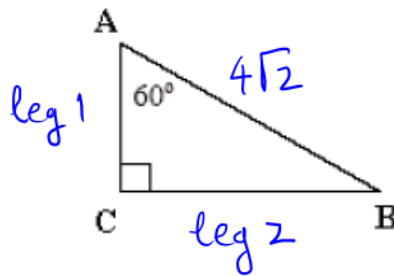
BC. short leg

$$x = \text{hypotenuse} : 2 \cdot 2\sqrt{2} = \boxed{4\sqrt{2}}$$

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Question 2:

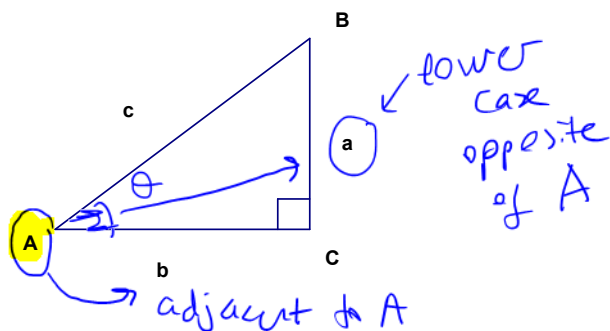
Right triangle ABC is given below. If $AB = 4\sqrt{2}$, find the area of this triangle.



$$\underline{\underline{\text{area} = \frac{1}{2} \cdot \text{leg 1} \cdot \text{leg 2}}}$$

Choices can be seen on the video.

The Six Trigonometric Functions of an Angle



$$\sin(A) = \frac{a}{c}$$

$$\cos(A) = \frac{b}{c}$$

Suppose we are given a right triangle, ABC where $\angle C = 90^\circ$. We define the trigonometric functions of either of the acute angles of the triangle as follows:

Sine Function:

$$\sin \theta = \frac{\text{length of the side opposite } \theta}{\text{length of hypotenuse}}$$

$$\frac{\text{opp.}}{\text{hyp.}}$$

Cosine Function:

$$\cos \theta = \frac{\text{length of the side adjacent to } \theta}{\text{length of hypotenuse}}$$

$$\frac{\text{adj}}{\text{hyp}}$$

Tangent Function:

$$\tan \theta = \frac{\text{length of the side opposite } \theta}{\text{length of the side adjacent to } \theta}$$

$$\frac{\text{opp}}{\text{adj}}$$

flip

Cotangent Function:

$$\cot \theta = \frac{\text{length of the side adjacent to } \theta}{\text{length of side opposite to } \theta}$$

$$\frac{\text{adj}}{\text{opp}}$$

Secant Function:

$$\sec \theta = \frac{\text{length of the hypotenuse}}{\text{length of the side adjacent to } \theta}$$

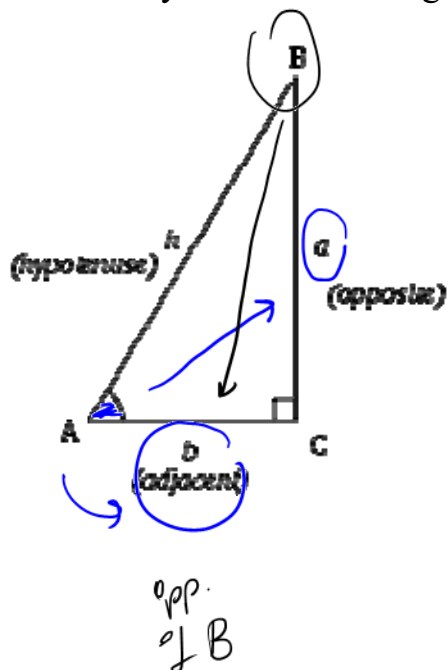
$$\frac{\text{hyp}}{\text{adj}}$$

Cosecant Function:

$$\csc \theta = \frac{\text{length of the hypotenuse}}{\text{length of the side opposite } \theta}$$

$$\frac{\text{hyp}}{\text{opp}}$$

Note: For acute angles the values of the trigonometric functions are always positive since they are ratios of lengths.



$$\sin(A) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{h}$$

$$\sin(B) = \frac{b}{h}$$

$$\cos(A) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{b}{h}$$

$$\tan(A) = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{b}$$

$$\cot(A) = \frac{\text{adjacent}}{\text{opposite}} = \frac{b}{a}$$

$$\sec(A) = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{h}{b}$$

$$\csc(A) = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{h}{a}$$

A useful mnemonic device:

SOH-CAH-TOA

$$S = \frac{O}{H}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

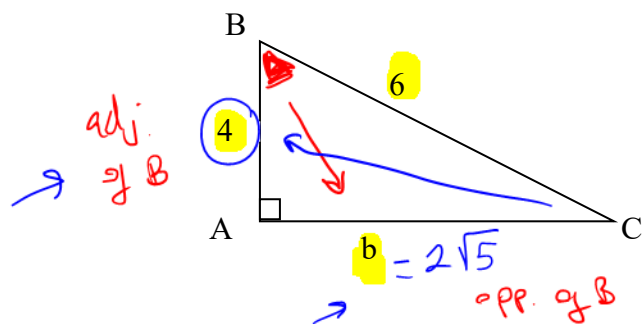
$$C = \frac{A}{H}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$T = \frac{O}{A}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

Example 1: Suppose you are given this triangle.



a. Find b .

$$b^2 + 4^2 = 6^2$$

$$b^2 + 16 = 36 \Rightarrow b^2 = 20 \Rightarrow b = \sqrt{20}$$

$$b = 2\sqrt{5}$$

b. Find the six trigonometric functions of angle B .

$$\sin(B) = \frac{\text{opp}}{\text{hyp}} = \frac{2\sqrt{5}}{6} = \frac{\sqrt{5}}{3} \quad \xrightarrow{\text{flip}} \csc(B) = \frac{3}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$$

$$\cos(B) = \frac{\text{adj}}{\text{hyp}} = \frac{4}{6} = \frac{2}{3} \quad \rightarrow \sec(B) = \frac{3}{2}$$

$$\tan(B) = \frac{\text{opp}}{\text{adj}} = \frac{2\sqrt{5}}{4} = \frac{\sqrt{5}}{2} \quad \rightarrow \cot(B) = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

c. Find the six trigonometric functions of angle C .

$$\sin(C) = \frac{4}{6} = \frac{2}{3} \quad \xrightarrow{\text{flip}}$$

$$\cos(C) = \frac{2\sqrt{5}}{6} = \frac{\sqrt{5}}{3} \quad \rightarrow$$

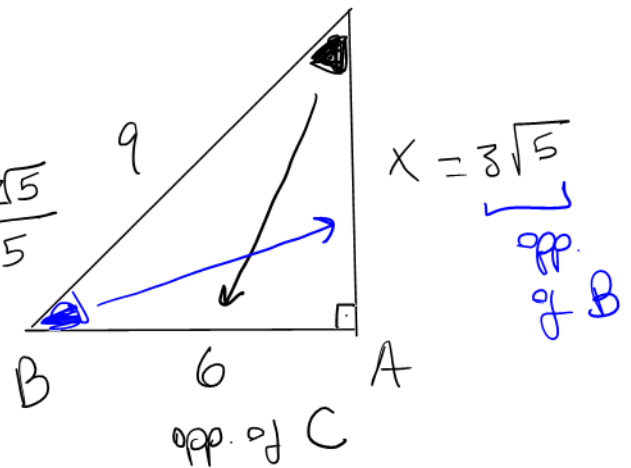
$$\tan(C) = \frac{4}{2\sqrt{5}} = \frac{2\sqrt{5}}{5} \quad \rightarrow$$

Example 2: Suppose that $\triangle BAC$ is a right triangle with $A = 90^\circ$. If $BC = 9$ and $AB = 6$, find each of the following:

$$\sin(C) = \frac{\text{opp}}{\text{hyp}} = \frac{6}{9} = \frac{2}{3}$$

$$\tan(C) = \frac{\text{opp}}{\text{adj}} = \frac{6}{3\sqrt{5}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\tan(B) = \frac{\text{opp}}{\text{adj}} = \frac{3\sqrt{5}}{6} = \frac{\sqrt{5}}{2}$$



$$x^2 + 6^2 = 9^2$$

$$x^2 + 36 = 81$$

$$\begin{array}{r} -36 \\ -36 \end{array}$$

$$x^2 = 45$$

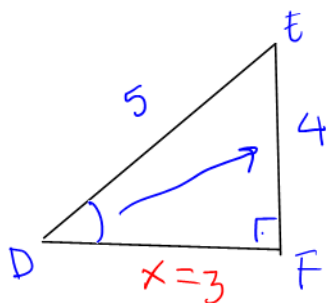
$$x = \sqrt{45} = 3\sqrt{5}$$

3-4-5 famous right triangle

typical

Example 3: Suppose that $\triangle DEF$ is a right triangle and D is an acute angle. If

$\sin D = \frac{4}{5}$, find $\sec(D)$ and $\tan(D)$.



$$x^2 + 4^2 = 5^2$$

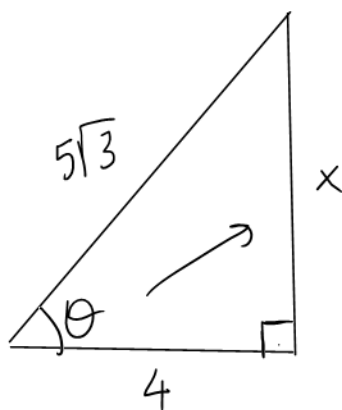
$$x^2 + 16 = 25 \Rightarrow x^2 = 9 \Rightarrow x = 3$$

$$\sin(D) = \frac{4}{5} = \frac{\text{opp}}{\text{hyp}}$$

$$\sec(D) = \frac{\text{hyp}}{\text{adj}} = \frac{5}{3}$$

$$\tan(D) = \frac{\text{opp}}{\text{adj}} = \frac{4}{3}$$

Example 4: Suppose that θ is an acute angle in a right triangle and $\sec \theta = \frac{5\sqrt{3}}{4}$. Find the $\cos \theta$.



$$\sec \theta = \frac{5\sqrt{3}}{4} = \frac{\text{hyp}}{\text{adj}}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{4}{5\sqrt{3}} = \frac{4\sqrt{3}}{15}$$

$$x^2 + 4^2 = (5\sqrt{3})^2$$

$$x^2 + 16 = 75$$

$$x^2 = 59$$

$$x = \sqrt{59}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{59}}{4}$$

famous right triangles: $\begin{cases} 3-4-5 \\ 5-12-13 \\ 8-15-17 \end{cases}$

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Question 3: In right triangle ABC with right angle C, $\sin A = \frac{5}{13}$. Find $\cot(A)$

Choices can be seen on the video.

Cofunctions

A special relationship exists between sine and cosine, tangent and cotangent, secant and cosecant. The relationship can be summarized as follows:

Function of θ = Cofunction of the complement of θ

Recall that two angles are **complementary** if their sum is 90° . In example 10, angles B and C are complementary, since $A = 90^\circ$. Notice that $\sin C = \cos B$ and $\sin B = \cos C$. The same will be true of any pair of cofunctions.

Cofunction relationships

$$\sin A = \cos(90^\circ - A)$$

$$\cos A = \sin(90^\circ - A)$$

$$\tan A = \cot(90^\circ - A)$$

$$\cot A = \tan(90^\circ - A)$$

$$\sec A = \csc(90^\circ - A)$$

$$\csc A = \sec(90^\circ - A)$$

$$x + y = 90^\circ$$

$$\sin x = \cos y$$

$$\cos x = \sin y$$

$$10^\circ + 80^\circ = 90^\circ$$

For example,

$$\sin 60^\circ = \cos 30^\circ;$$

$$\tan 10^\circ = \cot 80^\circ;$$

$$\csc 70^\circ = \sec 20^\circ.$$

Example: If $\sin(x) = \frac{1}{3}$, find

$$\cos(90^\circ - x) = \sin(x) = \frac{1}{3}$$