

## Section 6.1 - Sum and Difference Formulas

**In this section, we will learn some formulas about finding the trig values for the sum and difference of angles.**

For example; we know that  $\sin(30^\circ) = \frac{1}{2}$  and  $\sin(45^\circ) = \frac{\sqrt{2}}{2}$ .

What if we need to compute  $\sin(30^\circ + 45^\circ) = \sin(75^\circ)$ ?

$75^\circ$  is not one of the angles we covered on the unit circle. Can we compute its trig functions using known angles from the unit circle?

**Important:**  $\sin(A + B) \neq \sin(A) + \sin(B)$   
 $\cos(A + B) \neq \cos(A) + \cos(B)$

**To see this:**

$$\sin(30^\circ + 60^\circ) = \sin(90^\circ) = 1$$

$$\sin(30^\circ) + \sin(60^\circ) = \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1 + \sqrt{3}}{2}$$

$$\sin(30^\circ + 60^\circ) \neq \sin(30^\circ) + \sin(60^\circ)$$

### Sum and Difference Formulas for Sine, Cosine and Tangent

$$\sin(A + B) = \sin A \cos B + \sin B \cos A$$

$$\sin(A - B) = \sin A \cos B - \sin B \cos A$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

**Example 1:** Evaluate  $\sin(75^\circ)$  using sum and difference formulas.

**Hint:**  $75^\circ = 30^\circ + 45^\circ$

**Example 2:** Evaluate  $\sin(15^\circ)$  using sum and difference formulas.

**Hint:**  $15^\circ = 60^\circ - 45^\circ$

**Example 3: Evaluate**  $\cos\left(\frac{7\pi}{12}\right)$  **using sum and difference formulas.**

**Hint:**  $\frac{7\pi}{12} = \frac{\pi}{4} + \frac{\pi}{3}$

**Example 4: Evaluate  $\tan\left(\frac{\pi}{12}\right)$  using sum and difference formulas.**

**Hint:**  $\frac{\pi}{12} = \frac{\pi}{4} - \frac{\pi}{6}$

**Remark:** In most examples, the “hint” will not be given. You need to brainstorm a bit to see which two “EASY” angles can be used to get the given angle. Many times, there are many ways to do this, try to choose the easiest two angles from the unit circle.

**Examples in degrees:**

$$105^{\circ} = 60^{\circ} + 45^{\circ} \quad \text{or} \quad 105^{\circ} = 135^{\circ} - 30^{\circ}$$

$$15^{\circ} = 60^{\circ} - 45^{\circ} \quad \text{or} \quad 15^{\circ} = 45^{\circ} - 30^{\circ}$$

$$75^{\circ} = 30^{\circ} + 45^{\circ} \quad \text{or} \quad 75^{\circ} = 120^{\circ} - 45^{\circ}$$

**Examples in radians:**

$$\frac{\pi}{12} = \frac{\pi}{4} - \frac{\pi}{6} \quad \text{or} \quad \frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$$

$$\frac{5\pi}{12} = \frac{\pi}{4} + \frac{\pi}{6}$$

$$\frac{7\pi}{12} = \frac{\pi}{4} + \frac{\pi}{3}$$

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**Question#1: Use sum and difference formulas to evaluate:  $\cos(15^\circ)$**



**Example 5:** Given that  $\tan(A) = 2$  and  $\tan(B) = 6$ , evaluate  $\tan(A + B)$ .

**Example 6:** Given that  $\tan(x) = 5$ , evaluate  $\tan\left(x - \frac{\pi}{4}\right)$ .

**Example 7:** Given that  $\cos(x) = \frac{1}{3}$  and  $0 < x < \frac{\pi}{2}$ , evaluate  $\cos\left(x - \frac{\pi}{6}\right)$ .

**Example 8:** Suppose that  $\sin \alpha = \frac{3}{5}$  and  $\cos \beta = \frac{5}{13}$  where  $0 < \alpha, \beta < \frac{\pi}{2}$ .

Find each of these:

a.  $\sin(\alpha + \beta)$

b.  $\cos(\alpha - \beta)$

**Example 8:** Suppose  $\cos x = -\frac{15}{17}$  and  $\tan y = \frac{1}{2}$  where  $\pi < x, y < \frac{3\pi}{2}$ .

Find  $\cos(x + y)$ .

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**Question#2:** Given  $\sin(x) = \frac{4}{5}$  and  $0 < x < \frac{\pi}{2}$ , use sum and difference

**formulas to evaluate:**  $\sin\left(x + \frac{\pi}{4}\right)$

**REMARK:**

**We have used the following facts in previous sections:**

$$\sin(x + \pi) = -\sin x$$

$$\cos(x + \pi) = -\cos x$$

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$

**Now, we can prove these facts using sum and difference formulas.**

**Proofs:**

$$\sin(x + \pi) = \sin x \cos \pi + \sin \pi \cos x = \sin x \cdot (-1) + 0 \cdot \cos x = -\sin x$$

$$\cos(x + \pi) = \cos x \cos \pi - \sin x \sin \pi = \cos x \cdot (-1) - \sin x \cdot 0 = -\cos x$$

$$\sin\left(\frac{\pi}{2} - x\right) = \sin\left(\frac{\pi}{2}\right)\cos x - \sin x \cos\left(\frac{\pi}{2}\right) = 1 \cdot \cos x - \sin x \cdot 0 = \cos x$$

**Example 9:** Simplify each.

a)  $\sin(2x)\cos(15x) + \sin(15x)\cos(2x)$

b)  $\sin 10^\circ \cos 55^\circ - \sin 55^\circ \cos 10^\circ$

c)  $\cos\left(\frac{7\pi}{12}\right)\cos\left(\frac{\pi}{12}\right) + \sin\left(\frac{7\pi}{12}\right)\sin\left(\frac{\pi}{12}\right)$



**Example 10:** Simplify each.

a)  $\frac{\tan 40^\circ + \tan 5^\circ}{1 - \tan 40^\circ \tan 5^\circ}$

b)  $\frac{\tan 80^\circ - \tan 15^\circ}{1 + \tan 80^\circ \tan 15^\circ}$

**Example 11:** Let  $f(x) = \sin\left(x + \frac{\pi}{4}\right) - \sin\left(x - \frac{\pi}{4}\right)$ .

Simplify the formula for  $f(x)$  and find the minimum and maximum values.

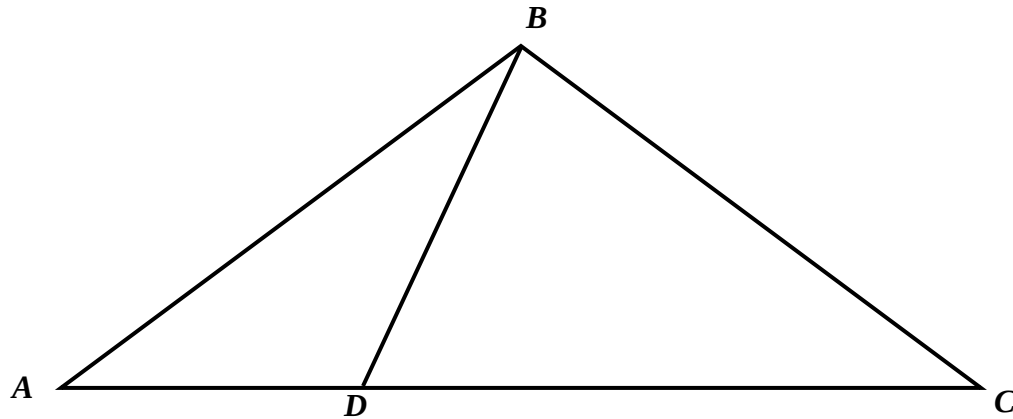
Find the  $x$  – intercepts of this function over the interval  $[0, 2\pi]$  .

Maximum Value: \_\_\_\_\_

Minimum Value: \_\_\_\_\_

$x$  – intercepts: \_\_\_\_\_

**Example 12:** In the figure below,  $\sin(\angle BAD) = \frac{1}{5}$  and  $\sin(\angle ABD) = \frac{3}{5}$ .  
Find  $\sin(\angle BDC)$ . (The image is not drawn to scale.)



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**Question#3: Given  $\sin(x) = \frac{4}{5}$  and  $\sin(y) = \frac{5}{13}$ , where  $0 < x, y < \frac{\pi}{2}$ , use sum and difference formulas to evaluate:  $\sin(x + y)$**