

# Math 4377

## Review Topics for In-Class Exam 1

### 1 Linear equations

#### 1.1 Systems of linear equations

Know how to solve a linear system over a field  $F$  by forming the augmented matrix and applying row operations.

#### 1.2 Row reduction and echelon forms

Does the solution set to a linear system change under elementary row operations? What are independent/free variables? How can we tell there are free variables by looking at the row-reduced echelon form?

#### 1.3 Vector equations

How can we rewrite a linear system  $Ax = b$  in vector form? Can we solve the system if  $b$  can be written as a linear combination of the column vectors of  $A$ ?

#### 1.4 Solution sets

How do the solutions to an inhomogeneous system relate to the solutions of the corresponding homogeneous one?

#### 1.5 Matrix inverse

Know how to compute the inverse of a matrix. What is the inverse of  $AB$  in terms of  $A^{-1}$  and  $B^{-1}$ ? What are the properties of the system  $Ax = b$  if  $A$  is invertible?

### 2 Vector spaces

What are the defining properties of a vector space? Know the difference between finite and infinite-dimensional vector spaces.

## 2.1 Subspaces

Know how to test whether a subset of a vector space is a subspace.

## 2.2 Spanning sets and linear independence

What is the span of a set of vectors? When are vectors linearly independent?

Does the span of the row vectors of a matrix  $A$  change under elementary row operations? What about the span of column vectors?

If  $Ax = 0$  has non-trivial solutions, what can we say about the column vectors of  $A$ ?

## 2.3 Bases and dimension

You can always extend a linearly independent set to a basis by adding appropriate vectors to it. What condition is needed when adding a vector to a linearly independent set to preserve linear independence?

Compute the dimension of  $W_1 + W_2$  for subspaces  $W_1$  and  $W_2$ . How many vectors are needed for a spanning set, how many can a linearly independent set have?

## 2.4 Coordinates and change of coordinates

How do we compute the change of coordinates matrix? How do we compute coordinates of a vector with respect to a new basis?

# 3 Linear Transformations

Tell the difference between linear and non-linear transformations. Know how to compare two linear transformations efficiently by invoking a basis.

## 3.1 Kernel and range, nullity and rank

What is the sum of nullity and rank? Describe the properties of a linear transformation with zero rank or with zero nullity.