

Math 4377
Advanced Linear Algebra
Fall 2008

Homework Set 7, due Tuesday, Oct 7, 1pm

Section 2.6

- 3 Consider the vectors $\alpha_1 = (-1, 0, 1, 2)$, $\alpha_2 = (3, 4, -2, 5)$, $\alpha_3 = (1, 4, 0, 9)$ in \mathbb{R}^4 . Find a system of homogeneous linear equations for 4 unknowns x_1, x_2, x_3, x_4 such that the solutions give all vectors (x_1, x_2, x_3, x_4) that are in the subspace spanned by α_1, α_2 and α_3 .
- 4 In \mathbb{C}^3 , let $\alpha_1 = (1, 0, -i)$, $\alpha_2 = (1+i, 1-i, 1)$, and $\alpha_3 = (i, i, i)$. Prove that these three vectors form a basis $B = \{\alpha_1, \alpha_2, \alpha_3\}$ for \mathbb{C}^3 . What are the coordinates of the vector $\gamma = (a, b, c)$ with respect to this (ordered) basis B ?
- x Consider another basis B' for \mathbb{C}^3 given by $\beta_1 = (1, i, i)$, $\beta_2 = (0, 1, i)$, and $\beta_3 = (0, 0, 1)$. (You do not have to prove it is a basis.) Find the change-of-coordinates matrix $P_B^{B'}$ from B to B' and compute the coordinate vector $[\gamma]_{B'}$ for the vector γ in the preceding problem.

Section 3.1

- 1 Which of the following functions from \mathbb{R}^2 to \mathbb{R}^2 are linear transformations? Explain why/why not.
- (a) $T(x_1, x_2) = (1 + x_1, x_2)$,
 - (b) $T(x_1, x_2) = (x_2, x_1)$,
 - (c) $T(x_1, x_2) = (x_1^2, x_2)$,
 - (d) $T(x_1, x_2) = (\sin x_1, x_2)$,
 - (e) $T(x_1, x_2) = (x_1 - x_2, 0)$.
- 3 Let V be the vector space of all polynomials over \mathbb{R} , meaning all functions of the form $p(x) = c_0 + c_1x + c_2x^2 + \cdots + c_nx^n$, $n \in \mathbb{N}$. Consider the linear map $T : V \rightarrow V$ which maps p to $p'(x) = c_1 + 2c_2x + \cdots + nc_nx^{n-1}$ (differentiation). Describe the range and the null space for this differentiation transformation. Do the same for the integration map $S : V \rightarrow V$ which maps p to $P(x) = \int_0^x p(s)ds$.
- 4 Is there a linear transformation T from \mathbb{R}^3 to \mathbb{R}^2 such that $T(1, -1, 1) = (1, 0)$ and $T(1, 1, 1) = (0, 1)$? If so, find such a T and give $T(a, b, c)$. If not, explain why it does not exist.