PRINTABLE VERSION

Practice Test 4

Question 1

Given $P(E^C) = 0.43$, $P(F) = 0.52$, and $P(E \cap F) = 0.18$. Find $P(E|F^C)$.

a) ○ 0.7500

b) ○ 0.9069

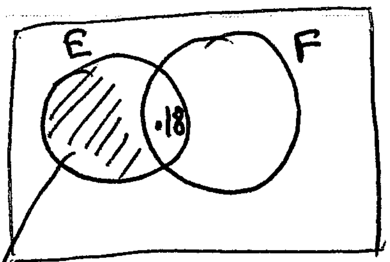
c) ○ 0.8125

d) ○ 0.5342

e) ○ 0.3461

f) ○ None of the above.

$$P(E|F^c) = \frac{P(E\cap F^c)}{P(F^c)} = \frac{0.39}{1-0.52} = \frac{0.39}{0.48} = \boxed{0.8125}$$



$P(E^c) = 0.43$

$P(E) = 1 - 0.43 = 0.57$

$P(E \cap F^c) = 0.57 - 0.18 = 0.39$

Question 2

Two cards are drawn without replacement from a well-shuffled deck of 52 playing cards. What is the probability that the first card drawn is a 2 and the second card drawn is a 3?

a) ○ 0.0059

b) ○ 0.1569

c) ○ 0.1554

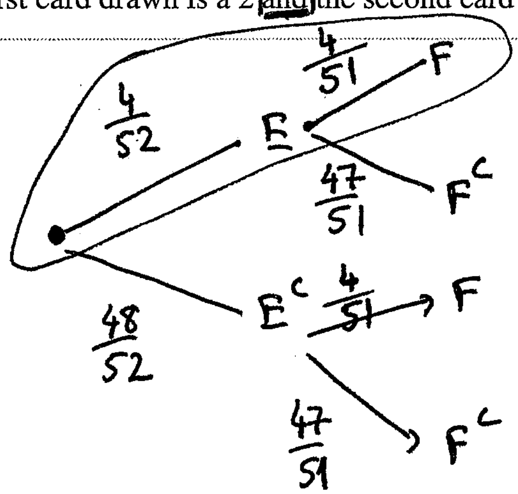
d) ○ 0.0045

e) ○ 0.0060

f) ○ None of the above.

E: 1st card is 2

F: 2nd card is 3



$$P(E\cap F) = \frac{4\cdot 4}{52\cdot 51} = \frac{16}{2652} = \boxed{0.0060}$$

Question 3

A pair of fair dice is cast. What is the probabiliy that at least one of the numbers falling uppermost is a 1, given that the sum of the numbers falling uppermost is even?

a) ○ 0.4545

b) ○ 0.3056

c) ○ 0.3667

d) ○ 0.2778

e) ○ 0.1389

f) ○ None of the above.

E: at least one number is 1

F: sum is even

$P(F) = \frac{18}{36}$

$P(E \mid F) = \frac{P(E \cap F)}{P(F)} = \frac{5/36}{18/36} = \frac{5}{18} = \boxed{0.2778}$

$E = \{(1,1), (1,2), (2,1), (1,3), (3,1), (1,4), (4,1), (1,5), (5,1), (1,6), (6,1)\}$

$E \cap F = \{(1,1), (1,3), (3,1), (1,5), (5,1)\}$

Question 4

A recording company obtains the blank CDs used to produce its labels from three compact disk manufacturers: I, II, and III. The quality control department of the company has determined that 7% of the compact disks produced by manufacturer I are defective, 5% of those produced by manufacturer II are defective, and 4% of those produced by manufacturer III are defective. Manufacturers I, II, and III supply 37%, 45%, and 18%, respectively, of the compact disks used by the company. What is the probability that a randomly selected label produced by the company will contain a defective compact disk?

a) ○ 0.1600

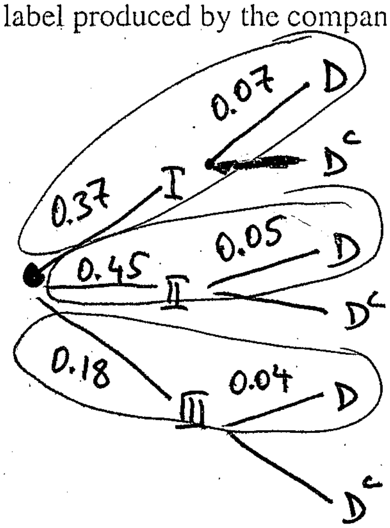
b) ○ 0.0556

c) ○ 0.0225

d) ○ 0.0072

e) ○ 0.0259

f) ○ None of the above.



$P(D) = P(D \cap I) + P(D \cap II) + P(D \cap III)$

$= (0.37)(0.07) + (0.45)(0.05) + (0.18)(0.04)$

$\boxed{= 0.0556}$

Question 5

There are three colored cookie jars. One jar is blue, another green and the last one pink. The blue jar contains 11 chocolate chip and 12 sugar cookies. The green jar contains 13 chocolate chip, 5 sugar and 14 peanut butter cookies. The pink jar contains 6 chocolate chip, 15 sugar and 8 peanut butter cookies. One of the three cookie jars is chosen at random. The probability that the blue jar, green jar, and pink jar will be chosen is 1/2, 1/4, and 1/4 respectively. A cookie is then chosen at random from the chosen jar. What is the probability that the pink jar was chosen, if it is known that the cookie was a sugar cookie?

a) ○ 0.5172

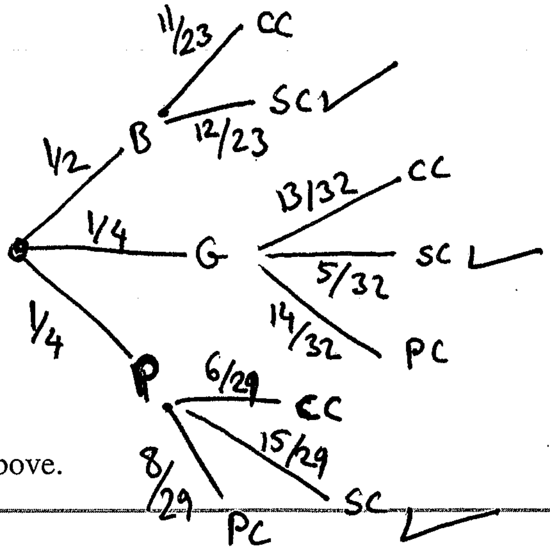
b) ○ 0.4328

c) ○ 0.3013

d) ○ 0.1293

e) ○ 0.1724

f) ○ None of the above.



$$P(P \mid SC) = \frac{P(P \cap SC)}{P(SC)}$$

$$= \frac{\frac{1}{4} \cdot \frac{15}{29}}{\frac{1}{4} \cdot \frac{15}{29} + \frac{1}{4} \cdot \frac{5}{32} + \frac{1}{2} \cdot \frac{12}{23}}$$

$$= \boxed{0.3013}$$

Question 6

John is interested in purchasing a multi-office building containing five offices. The current owner provides the following probability distribution indicating the probability that the given number of offices will be leased each year.

Number of Lease Offices	0	1	2	3	4	5
Probability	10/37	6/37	7/37	4/37	2/37	8/37

If each yearly lease is $12,000, how much could John expect to collect in yearly leases for the whole building in a given year?(in dollars)

a) ○ E(X) = $25,945.95

b) ○ E(X) = $26,045.95

c) ○ E(X) = $25,855.95

d) ○ E(X) = $26,035.95

e) ○ E(X) = $25,925.95

f) ○ None of the above.

$$E(X) = 0 \cdot \frac{10}{37} + 1 \cdot \frac{6}{37} + 2 \cdot \frac{7}{37} + 3 \cdot \frac{4}{37} + 4 \cdot \frac{2}{37} + 5 \cdot \frac{8}{37}$$

$$= \frac{80}{37}$$

$$\Rightarrow \text{Lease Money} = 12{,}000 \cdot \frac{80}{37} = 25{,}945.95$$

Question 7

A probability distribution has a mean of 28 and a standard deviation of 4. Use Chebychev's inequality to estimate the probability that an outcome of the experiment lies between 20 and 36.

a) ○ 0.2500

b) ○ 0.9796

$\mu = 28$

$\sigma = 4$

$$P(20 \le X \le 36) \ge 1 - \frac{1}{k^2}$$

c) 0.7500

d) 0.5000

e) 0.0204

f) None of the above.

$$P(\underbrace{\mu-k\sigma}_{20} \le X \le \underbrace{\mu+k\sigma}_{30}) \geq 1-\frac{1}{k^2}$$

$$28-4\cdot k=20 \Rightarrow k=2$$

$$\Rightarrow P(20 \le X \le 36) \geq 1-\frac{1}{4} = \boxed{0.7500}$$

Question 8

If the racetrack publishes that the odds in favor of a horse winning a race are 2 to 9, what is probability that the horse will not win the race?

a) 0.1818

b) 0.8182

c) 0.0556

d) 0.1111

e) 0.5000

f) None of the above.

Odds in favor 2 : 9

E: win the race

$$P(E) = \frac{2}{2+9} = \frac{2}{11}$$

$$\Rightarrow P(E^c) = 1-\frac{2}{11} = \frac{9}{11} = \boxed{0.8182}$$

Question 9

The probability distribution of a random variable X is given below.

x	1	4	6	8	9
$P(X = x)$	1/3	1/10	2/15	7/30	1/5

Given the mean

$$\mu = 5.20$$

Find the variance (Var(X)) and the standard deviation, respectively.

a) [42.00, 6.48]

b) [1136.00, 33.70]

c) [2099.89, 45.82]

d) [89.71, 9.47]

$$Var(X) = \frac{1}{3}(1-5.20)^2 + \frac{1}{10}(4-5.20)^2 + \frac{2}{15}(6-5.20)^2 + \frac{7}{30}(8-5.20)^2 + \frac{1}{5}(9-5.20)^2$$

$$= 10.83$$

$$\sigma(X) = \sqrt{Var(X)} = 3.29$$

e) [10.83, 3.29]

f) None of the above.

Question 10

Consider the following binomial experiment. The probability that a green jelly bean is chosen at random from a large package of jelly beans is 1/8. Sally chooses 13 jelly beans, what is the probability that at most 2 will be green jelly beans?

a) 0.7781

b) 0.5000

c) 0.2800

d) 0.7841

e) 0.2159

f) None of the above.

E: At most 2 G :

2G, 11 other
1G, 12 other
0G, 13 other

$p = \frac{1}{8}$, $n = 13$

$q = \frac{7}{8}$

$$P(E) = C(13,2)\left(\frac{1}{8}\right)^2\left(\frac{7}{8}\right)^{11} + C(13,1)\left(\frac{1}{8}\right)^1\left(\frac{7}{8}\right)^{12} + C(13,0)\left(\frac{1}{8}\right)^0\left(\frac{7}{8}\right)^{13}$$

$$= \boxed{0.7841}$$

Question 11

Consider the following binomial experiment. A survey shows 56% of households in Centercity own a DVD player. In a random sample of 19 households in this city, what is the probability that exactly 13 households own a DVD player?

a) 0.1045

b) 0.1051

c) 0.1046

d) 0.1050

e) 0.1048

f) None of the above.

Own a DVD : $p = 0.56$

$q = 0.44$

$n = 19$

P(exactly 13 own DVD)

$$= C(19,13)\,(0.56)^{13}\cdot(0.44)^{6} = \boxed{0.1048}$$

Question 12

Consider the following binomial experiment. A company owns 14 copiers. The probability that on a given day any one copier will break down is 3/25. Find the mean number of copiers that will break down on any given day.

a) 1.16

b) 1.40

c) 0.16

d) 1.68

e) 1.20

f) None of the above.

$n = 14$

$p = 3/25$

$\mu = n \cdot p = 14 \cdot \frac{3}{25} = 1.68$

Question 13

Let Z be a standard normal variable. Find P(Z > 2.23).

a) 0.9871

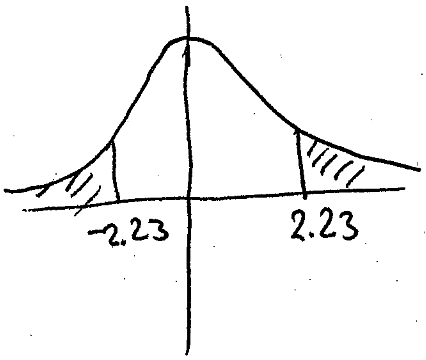
b) 0.0129

c) 0.9861

d) 0.0139

e) 0.0228

f) None of the above.



By symmetry

$P(Z > 2.23) = P(Z < -2.23)$

$= \boxed{0.0129}$

Question 14

Let Z be a standard normal variable. Find P(1.21 < Z < 3.00).

a) 0.8836

b) 0.1574

c) 0.1138

d) 0.8875

e) 0.1118

$P(1.21 < Z < 3.00)$

$= P(Z < 3.00) - P(Z < 1.21)$

$= 0.9987 - 0.8869$

$= \boxed{0.1118}$

f) ○ None of the above.

Question 15

Let Z be a standard normal variable. Find the value of z if z satisfies P(-z < Z < z) = 0.9936.

a) ○ 2.49

$P(Z<z) = \frac{1}{2}\left(P(-z<Z<z)+1\right)$

b) ○ 2.73

$= \frac{1}{2}(0.9936+1)$

c) ○ 2.70

d) ○ -2.40

$= 0.9968 \Rightarrow \boxed{z = 2.73}$

e) ○ -2.74

f) ○ None of the above.

Question 16

Let Z be a standard normal variable. Find the value of z if z satisfies P(Z < z) = 0.9970.

a) ○ 3.00

$\boxed{z = 2.75}$

b) ○ -2.69

c) ○ 2.75

d) ○ 2.70

e) ○ -2.75

f) ○ None of the above.

Question 17

Suppose X is a normal random variable with

$\mu = 40$

and

$\sigma = 20$

Find P(X > -11.4).

$P(X > -11.4) = P\left(Z > \frac{-11.4-40}{20}\right) = P(Z > -2.57)$

a) ○ 0.0062

$= 1 - P(Z \le -2.57) = 1 - 0.0051$

b) 0.9867

c) 0.0051

d) 0.9949

e) 0.9938

f) None of the above.

$$P(X > -11.4) = P\left(Z > \frac{-11.4-40}{20}\right) = P(Z > -2.57)$$
$$= 1 - P(Z \leq -2.57) = 1 - 0.0051$$
$$= 0.9949$$

Question 18

Endure All, a manufacturer of batteries claims that the lifetime of their batteries is normally distributed with a mean of 500 hours and a standard deviation of 40 hours What is the probability that an Endure All battery selected at random will last more than 575 hours?

a) 0.0359

b) 0.9699

c) 0.0307

d) 0.0301

e) 0.9693

f) None of the above.

$\mu = 500$
$\sigma = 40$

$$P(X > 575) = P\left(Z > \frac{575-500}{40}\right)$$
$$= P(Z > \underbrace{1.875}_{1.88})$$
$$= 0.9699$$

Question 19

Use the normal distribution to approximate the following binomial distribution. A fair coin is tossed 130 times. What is the probability of obtaining between 55 and 69 tails, inclusive? means $\leq$ $\geq$

a) 0.7494

b) 0.7134

c) 0.7221

d) 0.7523

e) 0.7179

f) None of the above.

$n = 130$
$p = \frac{1}{2}$
$\mu = 65 = \frac{1}{2} \cdot 130$
$\sigma = \sqrt{130 \cdot \frac{1}{2} \cdot \frac{1}{2}} = 5.70$

$$P(55 \leq X \leq 69) \approx P(54.5 \leq Y \leq 69.5)$$
$$= P\left(\frac{54.5-65}{5.70} \leq Z \leq \frac{69.5-65}{5.70}\right)$$
$$= P(-1.84 \leq Z \leq 0.79)$$
$$= P(Z \leq 0.79) - P(Z \leq -1.84)$$
$$= 0.7852 - 0.0329 = \boxed{0.7523}$$

Question 20

Use the normal distribution to approximate the following binomial distribution. A convenience store owner claims that 55% of the people buying from her store, on a certain day of the week, buy coffee during their visit. A random sample of 35 customers is made. If the store owner's claim is correct, what is the probability that fewer than 22 customers in the sample buy coffee during their visit on that certain day of the week?

people buy from store : $p = 0.55$

$q = 0.45$

$n = 35$

a) ○ 0.7967

b) ○ 0.8413

c) ○ 0.8238

d) ○ 0.7764

e) ○ 0.7224

f) ○ None of the above.

$$P(x < 22) \cong P(y \le 21.5)$$

$$= P\left(Z \le \frac{21.5 - 19.25}{2.94}\right)$$

$$= P(Z \le 0.76)$$

$$= 0.7764$$

$\mu = np = 35(0.55) = 19.25$

$\sigma = \sqrt{npq} = 2.94$