Question Find the domain and range of $\ln(\left|x^2 + y + 3z^3\right| + 1)$

Solution

Domain ln is defined for all positive numbers. For any values of x, y, z, $\left|x^2 + y + 3z^3\right|$ is always positive and so $\left|x^2 + y + 3z^3\right| + 1$ is also always positive. Thus, the domain is all x, y, z.

Range $\left|x^2 + y + 3z^3\right|$ is always positive, so $\left|x^2 + y + 3z^3\right| + 1 \geq 1$ Since ln is increasing, $\ln(\left|x^2 + y + 3z^3\right| + 1) \geq \ln(1) = 0$. Thus the range is $(0, \infty)$.

DOMAIN $\mathbb{R}^3$

RANGE $(0, \infty)$

Question Find the domain and range for $\ln(xy)$

Solution

Domain ln is defined for all positive numbers and for positive numbers only. Therefore we want xy to be positive. This happens if and only if $x > 0$ & $y > 0$ or $x < 0$ & $y < 0$. This means that the domain of definition is the first and the third quadrant.

Range In the first and third quadrant, xy is positive, but it can get very close to 0. For these values of xy, ln gets very small (close to $-\infty$). xy can also get very large, and for these values ln gets close to $+\infty$. Thus the range is $(-\infty, \infty)$.

DOMAIN 1^{st} and 3^{rd} Quadrants

RANGE $(-\infty, \infty)$