

**Solutions to Quiz 5**  
**Spring 2008 MATH 2433**

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**INSTRUCTIONS** You have 10 minutes for this quiz. Do not write on the back of the sheet, work on the back will not be graded.

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Let  $\mathbf{r}(t) = 2t\mathbf{i} + t^2\mathbf{j} + \ln t\mathbf{k}$  be the vector denoting the position of a moving point. Find

1. **(4 points)** The curvature of the path in terms of  $t$   
 $\mathbf{r}'(t) = \mathbf{i} + 2t\mathbf{j} + \frac{1}{t}\mathbf{k}$ ,  $\mathbf{a}(t) = 0\mathbf{i} + 2\mathbf{j} - \frac{1}{t^2}\mathbf{k}$

$$\frac{ds}{dt} = \|v\| = \sqrt{1 + 4t^2 + \frac{1}{t^2}} = \frac{\sqrt{4t^4 + t^2 + 1}}{t}$$

$$v \times a = \mathbf{b} = -\frac{4}{t}\mathbf{i} + \frac{1}{t^2}\mathbf{j} + 2\mathbf{k}$$

so

$$\kappa = \frac{|v \times a|}{\|v\|^3} = \frac{t\sqrt{4t^4 + 16t^2 + 1}}{(4t^4 + t^2 + 1)^{3/2}}$$

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2. **(4 points)** The tangential component of acceleration

$$a_T = \frac{d^2s}{ds^2} = \frac{4t^4 - 1}{t^2\sqrt{4t^4 + t^2 + 1}}$$

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3. **(4 points)** The normal component of acceleration

$$a_N = \kappa \left(\frac{ds}{dt}\right)^2 = \|v\|^2 = \frac{1}{t}\sqrt{\frac{4t^4 + 16t^2 + 1}{4t^4 + t^2 + 1}}$$

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