Sulutions to Quiz 8
Spring 2008 MATH 2433

Name:——————————————————— PS Id:—————

INSTRUCTIONS You have 10 minutes for this quiz. Do not write on the back of the sheet, work on the back will not be graded. Show all your work. Full credit will not be given if either the work is completely missing or is incorrect.

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1. **(4 points)** Find $\frac{\partial u}{\partial s}$ and $\frac{\partial u}{\partial t}$ for
$$u = x^2 - xy + z^2$$
where
$$x = s\cos t, y = \sin(t-s), z = t\sin s$$
Solution
$$\frac{\partial u}{\partial s} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial u}{\partial z}\frac{\partial z}{\partial s} = (2x-y)\cos t + x\cos(t-s) + 2zt\cos(s)$$
$$= 2s\cos^2 t - \sin(t-s)\cos t + s\cos t\cos(t-s) + 2t^2\sin s\cos s$$
$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial u}{\partial z}\frac{\partial z}{\partial t} = (2x-y)(-s\sin t) - x\cos(t-s) + 2z\sin s$$
$$= -2s^2\cos t\sin t + s\sin(t-s) + 2t\sin^2 s$$

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2. **(4 points)** Show that the sphere $x^2+y^2+z^2-8x-8y-6z+24=0$ is tangent to the ellipsoid $x^2+3y^2+2z^2=9$ at the point $(2,1,1)$
Solution. Let us call the sphere f and the ellipsoid g.
$$\nabla f = (2x-8)\mathbf{i} + (2y-8)\mathbf{j} + (2z-6)\mathbf{k}$$
Therefore $\nabla f(2,1,1) = (-4,-6,-4)$. Similarly,
$$\nabla g = 2x\mathbf{i} + 6y\mathbf{j} + 4z\mathbf{k}$$
$\nabla g(2,1,1) = (4,6,4)$. Thus $\nabla f(2,1,1) = -\nabla g(2,1,1)$. Therefore the normal vectors at $(2,1,1)$ are parallel. Hence the surfaces are tangent at $(2,1,1)$

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3. **(4 points)** State whether TRUE or FALSE, briefly giving reasons
   (a) The equation of the tangent plane to f at the point $\mathbf{x_0}$ is $\nabla f(\mathbf{x_0})(\mathbf{x}-\mathbf{x_0})$.
   Solution TRUE
   (b) The vector $t = \frac{\partial f}{\partial y}(x_0,y_0)\mathbf{i} + \frac{\partial f}{\partial x}(x_0,y_0)\mathbf{j}$ is perpendicular to the gradient of f at (x_0,y_0).
   Solution FALSE. The dot product with the gradient is not 0.
   (c) If $u = u(x,y)$ is continuously differentiable and y is a differentiable function of x that satisfies $u(x,y)=0$, then at all points (x,y) where $\partial u/\partial y \neq 0$, $\frac{dy}{dx} = -\frac{\partial u/\partial x}{\partial u/\partial y}$
   Solution TRUE
   (d) Up to multiplication by a scalar constant, the tangent vector and the normal vector to a three dimensional surface are unique.
   Solution FALSE. The normal vector is unique, however, we can find a tangent PLANE to the surface at each point so the tangent vector cannot be unique up to multiplication by scalar.