1. **16.7 # 44** Integrate $f(x, y, z) = x^2y^2$ over the solid bounded above by the cylinder $y^2 + z = 4$, below by $y + z = 2$ and on the sides by the planes $x = 0$ and $x = 2$.

   The planes $x = 0$ and $x = 2$ give you the x-limits of integration for free. To get the y-limits, set $y^2 + z = 4$ equal to $y + z = 2$ to get

   $$y^2 - 4 = y - 2$$

   from which you get

   $$y^2 - y - 2 = 0 \implies y = -1, 2.$$

   Hence the integral is

   $$\int_0^2 \int_{-1}^2 \int_{2-y^2}^{4-y^2} x^2y^2 \text{ d}x \text{ d}y \text{ d}z.$$

   Now solve.

2. **16.7 # 48** Find the volume of the solid bounded by the paraboliods $z = 2 - x^2 - y^2$ and $z = x^2 + y^2$.

   Set the two curves equal to get

   $$2 - x^2 - y^2 = x^2 + y^2 \implies x^2 + y^2 = 1.$$

   This tells us that that we should be using polar co-ordinates. θ goes from $[0, 2\pi]$ and r from 0 to 1. The volume therefore is

   $$\int_{\theta=0}^{2\pi} \int_{r=0}^{1} \int_{z=r^2}^{2-r^2} r \text{ d}z \text{ d}r \text{ d}\theta = 2 \int_{\theta=0}^{2\pi} \int_{r=0}^{1} (r - r^3) \text{ d}r \text{ d}\theta = \pi.$$

3. **16.8 # 23** As done for # 22 in the lab, set $x^2 + y^2 = r^2$ and solve the quadratic to get $r = \frac{1}{2}$. Changing the problem to one involving polar co-ordinates, we get

   $$\int_{\theta=0}^{2\pi} \int_{0}^{\frac{1}{2}} \int_{r\sqrt{3}}^{\sqrt{1-r^2}} r \text{ d}z \text{ d}r \text{ d}\theta = \text{ some amount of work } = \frac{1}{3}\pi(2 - \sqrt{3}).$$