

HW 1

Please, write clearly and justify your arguments using the theory covered in class to get credit for your work.

(1) [3Pts] Prove that

$$\sum_{i=1}^n i^2 = \frac{1}{6}n(n+1)(2n+1) \quad n \in \mathbb{N}.$$

(2) [3Pts] Prove that, for any  $n \in \mathbb{N}$ , the number  $9^n - 4^n$  is divisible by 5.

(3) [3Pts] Prove that, for any  $n \geq 4$  the following inequality holds

$$n^2 \leq 2^n.$$

(4) [4 Pts]

Let  $R$  be the relation on  $\mathbb{Z}$  defined as follows:

*For  $a, b \in \mathbb{Z}$ ,  $aRb$  if and only if  $a$  is a multiple of  $b$*

- (a) Is  $R$  reflexive?
- (b) If  $R$  symmetric?
- (c) Is  $R$  transitive?
- (d) Is  $R$  and equivalence relation?

For each question, prove it or disprove it using a counterexample.