

HW 7

Please, write clearly and justify all your statements using the material covered in class to get credit for your work.

(1) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be given by

$$f(x) = \begin{cases} \sin(1/x) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0. \end{cases}$$

Show that f is not continuous at $x = 0$.

(2) Let

$$f(x) = \begin{cases} \frac{x^2+4x-21}{x-3} & \text{if } x \neq 3 \\ a & \text{if } x = 3. \end{cases}$$

Define a so that f will be continuous at $x = 3$.

(3) Determine a condition (a bound independent on x) on $|x - 1|$ such that

(a) $|x^2 - 1| < 1/2$.

(b) $|x^2 - 1| < 0.01$.

(4) Let $f : D \rightarrow \mathbb{R}$ and c be an accumulation point of D . Suppose that $\lim_{x \rightarrow c} f(x) = L$.

(a) Prove that $\lim_{x \rightarrow c} |f(x)| = |L|$.

(b) If $f(x) \geq 0$ for all $x \in D$, prove that $\lim_{x \rightarrow c} \sqrt{f(x)} = \sqrt{L}$.