Math 3333 Name:

HW 2

Please, write clearly and justify your arguments using the theory covered in class to get credit for your work.

(1) [5Pts] Let S be a nonempty and bounded subset of $\mathbb{R}$. Prove that $m = \inf S$ is unique.

Proof. Since the set is nonempty and bounded, it has a lower bound and an infimum m. Suppose that there exists another number $m_1 = \inf S$ with $m_1 \neq m$. Then either $m_1 > m$ or $m_1 < m$. If $m_1 > m$ then m would not be $\inf S$ *since it could not be the greatest lower bound of S. Similarly, if $m_1 < m$ then m_1 would not be* $\inf S$ *since it could not be the greatest lower bound of S. Thus it must be $m = m_1$*

(2) [5Pts] Let $S \subset \mathbb{R}$ be nonempty. Show that S is bounded if and only if there exists a closed bounded interval I such that $S \subset I$.

Proof. Since the set is nonempty and bounded, it has upper and lower bounds, a supremum M and an infimun m. It follows that, for any $x \in S$, $x \leq M$ and $x \geq m$. Thus S is contained in the interval $I = [m, M]$.

Conversely, suppose that $S \subset I = [a, b]$, where $a, b \in \mathbb{R}$. It follows that $a \leq \inf S$ and $\sup S \leq b$. *Hence S is bounded.*