**Math 3333** Name:

# Quiz/HW 3

Please, write clearly and justify your steps to get credit for your work.

[10 Pts] Mark each statement as True or False. If False, show a counter-example. If True, justify your answer.

(a) Every non-empty open set contains at least two points.

TRUE. If a non-empty set contains a single point it is necessarily a closed set. Hence a non-empty open set must contain at least 2 points

(b) The set $S = \{\frac{1}{n} : n \in \mathbb{N}\}$ is closed.

FALSE. 0 is a boundary point of $S$ that does not belong to $S$.

(c) If $S$ is unbounded then $S$ has an accumulation point.

FALSE. $\mathbb{N}$ is unbounded and has no accumulation points since each point is isolated.

(d) If $S \subset \mathbb{R}$ is open and $x$ is an accumulation point of $S$, then $x \in S$.

FALSE. Let $S = (0, 1)$. Then $x = 0$ is an accumulation point of $S$ but $x \notin S$.

(e) If $S \subset \mathbb{R}$ is a closed, then there is at least one point in $\mathbb{R}$ that is an accumulation point of $S$.

FALSE. Let $S = \{1\}$. This set is closed but has no accumulation points.