**Math 3333** Name: SOLUTION

## Quiz 6

Please, write clearly and justify all your statements using the material covered in class to get credit for your work.

(1)[ 5pts] Find an example of a sequence $(s_n)$ of real numbers satisfying each of the following properties. .

(a) Cauchy, but not monotone.

$s_n = (-1)^n \frac{1}{n}$

(b) Monotone, but not Cauchy.

$s_n = n$

(c) Bounded, but not Cauchy.

$s_n = (-1)^n$

(d) Monotone sequence $(s_n)$ without a convergent subsequence.

$s_n = n$

(e) $(s_n)$ converges to 0 but $\sum s_n$ is not a convergent series.

$s_n = \frac{1}{n}$

(2)[5pts] Let $(a_n)$ be a sequence of nonnegative real numbers. Prove that $\sum a_n$ converges iff the sequence of partial sums is bounded.

Proof. Since $a_n \geq 0$ for all $n$, then the sequence of partial sums $(s_n) = (\sum_{k \leq n} a_k)$ is monotone nondecreasing.

It follows by the Monotone Convergence Theorem of sequences that $(s_n)$ converges iff it is bounded.