Math 3333 Name: SOLUTION

Quiz 7

Please, write clearly and justify all your statements to get credit for your work.

(1a)[2 Pts] Let

$$h(x) = \frac{x^2 + 3x + 2}{x + 2}$$

Is h continuous at $x = -2$? Justify your answer.

Since the denominator of $h(x)$ vanishes at $x = 0$, then h is not defined at $x = 0$ and, hence, it is not continuous at 0.

(1b)[2 Pts] Let

$$f(x) = \begin{cases} \frac{x^2+3x+2}{x+2} & \text{if } x \neq -2 \\ 0 & \text{if } x = -2. \end{cases}$$

Prove that f is discontinuous at $x = -2$.

We observe that

$$\lim_{x \to -2} f(x) = \lim_{x \to -2} \frac{x^2 + 3x + 2}{x + 2} = \lim_{x \to -2} \frac{(x + 1)(x + 2)}{x + 2} = -1.$$

Since $0 = f(-2) \neq \lim_{x \to -2} f(x)$, then f is discontinuous at $x = -2$.

(1c)[2 Pts] Define a so that g below will be continuous at $x = -2$. Prove the continuity at at $x = -2$.

$$g(x) = \begin{cases} \frac{x^2+3x+2}{x+2} & \text{if } x \neq -2 \\ a & \text{if } x = -2. \end{cases}$$

It follows from part (b) that if we set $a = g(-2) = -1$, then $\lim_{x \to -2} g(x) = -1 = g(-2)$. Hence we need to assign $a = g(-2) = -1$ to ensure that g is continuous at $x = -2$.

(2)[2 Pts] Determine a bound δ independent of x such that $|x - 2| < \delta$ implies

$$|x^2 - 4| < 0.01.$$

Write $|x^2-4| = |x-2||x+2|$. If $|x-2| < 1$, then $|x| < 3$ and $|x^2-4| = |x+2||x-2| < (|x|+2)|x-2| < 5|x - 2|$.

Hence, to have $|x^2 - 4| < \epsilon$, it is sufficient to require that $5|x - 2| < \epsilon$ and $|x - 2| < 1$. Let $\delta = \min(1, \epsilon/5)$. Then, if $|x - 2| < \delta$ it follows that $|x^2 - 4| < \epsilon$.

Thus, we can choose $\delta < \frac{1}{5}0.01$; that is, $|x - 2| < \frac{1}{5} \cdot 0.01$ implies that $|x^2 - 4| < 0.01$.