Math 4310 – Fall 2024 Name: SOLUTION

HW #2

(7.3.1)

We compare rugby plater (r) vs multisport players (m)

We test $H_0 : \mu_r \leq \mu_m$ against $H_1 : \mu_r > \mu_m$ with $\alpha = 0.01$.

Data: $n_r = 24$, $\bar{x}_r = 27.75$, $s_r = 2.64$; $n_m = 40$, $\bar{x}_m = 22.41$, $s_m = 1.27$. The sample variance is $s_p^2 = \frac{(n_r-1)s_r^2+(n_m-1)s_m^2}{n_r+n_m-2} = 3.600$.

Test statistic (Student t pdf):

$$t = \frac{\bar{x}_r - \bar{x}_m}{\sqrt{\frac{s_p^2}{n_r} + \frac{s_p^2}{n_m}}} = \frac{27.75 - 22.41}{\sqrt{\frac{3.600}{24} + \frac{3.600}{40}}} = 10.90$$

Rejection region: $t > t_{0.01;63} = 2.390$

Since $t > t_{0.005;63} = 2.390$, then $\underline{H_0 \text{ is REJECTED}}$.

NOTE: If one assumes that σ_r and σ_m are known and uses the Normal distribution, then Rejection region: $z > z_{0.01} = 2.326$ and the decision is the same.

(7.3.3)

We compare male patients with obstructive sleep apnea syndrome (o) and healthy male subjects (c).

We test $H_0 : \mu_o = \mu_c$ against $H_1 : \mu_o \neq \mu_c$ with $\alpha = 0.01$.

Data: $n_o = 26$, $\bar{x}_o = 111.060$, $s_o^2 = 48.398$; $n_c = 37$, $\bar{x}_c = 95.854$, $s_c^2 = 31.237$. The sample variance is $s_p^2 = \frac{(n_o-1)s_o^2+(n_c-1)s_c^2}{n_o+n_c-2} = 38.270$.

Test statistic (Student t pdf):

$$t = \frac{\bar{x}_o - \bar{x}_c}{\sqrt{\frac{s_p^2}{n_o} + \frac{s_p^2}{n_c}}} = \frac{111.060 - 95.854}{\sqrt{\frac{38.270}{26} + \frac{38.270}{37}}} = 9.61$$

Rejection region: $t > t_{0.005;62} = 2.660$ or $t < -t_{0.005;62} = -2.660$

Since $t > t_{0.005;63} = 2.660$, then $\underline{H_0 \text{ is REJECTED}}$.

(7.4.3)

We compare max pain intensity with metadone (m) or placebo (p). We consider the paired differences $d_i - p_i - m_i$

Pair t-test. We test $H_0 : \mu_d \leq 0$ against $H_1 : \mu_d > 0$ with $\alpha = 0.05$.

Data: $n = 11$, $\bar{d} = \frac{1}{11}\sum_{i=1}^{11} d_i = 9.618$, $s_d^2 = 102.204$.
Test statistic (Student t pdf):

$$t = \frac{\bar{d} - \mu_d}{s_{\bar{d}}} = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}} = \frac{9.618}{\sqrt{\frac{102.204}{11}}} = 3.155$$

Rejection region: $t > t_{0.05;10} = 1.812$
Since $t > t_{0.05;10} = 1.812$, then $\underline{H_0 \text{ is REJECTED}}$.

(7.5.1)

We examine proportions of gynecologists-obstetricians in the Flanders who performed at least one cesarean section

We test $H_0 : p \geq 0.35$ against $H_1 : p < 0.35$ with $\alpha = 0.05$.
Data: $n = 295$, $x = 90$, $\hat{p} = \frac{90}{295} = 0.305$.
Test statistic (Standard Normal pdf):

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.305 - 0.350}{\sqrt{\frac{(0.35)(0.65)}{295}}} = -1.621$$

Rejection region: $z < -z_{0.95} = -1.645$
Since $z > -z_{0.95} = -1.645$, then $\underline{H_0 \text{ is not REJECTED}}$.

(7.6.2)

We examine rates of posttraumatic stress disorder (PTSD) in mothers (m) and fathers (f).

We test $H_0 : p_f \geq p_m$ against $H_1 : p_f < p_m$ with $\alpha = 0.05$.
Data: $n_f = 175$, $x_f = 28$, $\hat{p}_f = 0.160$; $n_m = 180$, $x_m = 43$, $\hat{p}_m = 0.239$;
Test statistic (Standard Normal pdf):

$$z = \frac{\hat{p}_f - \hat{p}_m}{\hat{\sigma}_{\hat{p}_f - \hat{p}_m}}$$

where

$$\bar{p} = \frac{x_n + x_f}{n_m + n_f} = \frac{71}{355} = 0.200; \ \hat{\sigma}_{\hat{p}_f - \hat{p}_m} = \sqrt{\frac{\bar{p}(1-\bar{p})}{n_f} + \frac{\bar{p}(1-\bar{p})}{n_m}} = 0.042$$

Hence $z = (0.160 - 0.239)/0.042 = -1.667$
Rejection region: $z < -z_{0.95} = -1.645$
Since $z < -z_{0.95} = -1.645$, then $\underline{H_0 \text{ is REJECTED}}$.

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Quiz #2

(1) *Subjects in a Body Mass Index (BMI) study included a sample of 7 male soccer players and a sample of 8 male rugby players whose measured BMI was*

Soccer players: $23.5, 22.8, 23.2, 23.6, 22.9, 23.1, 23.6$

Rugby players: $23.1, 23.8, 24.6, 23.9, 24.9, 24.3, 24.1, 23.6$

Assume each population sample is normally distributed Is there sufficient evidence for one to claim that, in general, soccer players have a lower BMI than rugby players? Assume that the unknown variances are the same and set $\alpha = 0.005$. *You must state the hypothesis testing problem and indicate what type of test you use.*

Let μ_s and μ_r be the means of the BMI distribution of soccer and rugby players respectively

We test $H_0 : \mu_r = \mu_s$ against $H_1 : \mu_r \neq \mu_s$ with $\alpha = 0.01$.

Using R:

```
> x<-c(23.5, 22.8, 23.2, 23.6, 22.9, 23.1, 23.6)
> y<-c(23.1, 23.8, 24.6, 23.9, 24.9, 24.3, 24.1, 23.6)
> t.test(x,y,alternative = "less", paired = FALSE,var.equal = TRUE)

Two Sample t-test

data:  x and y
t = -3.2309, df = 13, p-value = 0.003283
alternative hypothesis:  true difference in means is less than 0
95 percent confidence interval:
-Inf -0.3590786
sample estimates:
mean of x mean of y
23.24286 24.03750
```

Since $p - value = 0.003283 < 0.005$, then $\underline{H_0 \text{ is REJECTED}}$.

(2) *A study compares the rates of posttraumatic stress disorder (PTSD) in mothers (m) and fathers (f). In this study, 28 out of 175 fathers and 43 out of 180 mothers were found to meet the criteria for PTSD. Is there enough evidence to conclude that mothers are more likely to develop PTSD than fathers; use* $\alpha = 0.01$. *You must state the hypothesis testing problem and indicate what type of test you use.*

We test $H_0 : p_m \leq p_f$ against $H_1 : p_m > p_f$ with $\alpha = 0.05$.

Data: $n_f = 175$, $x_f = 28$, $\hat{p}_f = 0.160$; $n_m = 180$, $x_m = 43$, $\hat{p}_m = 0.239$;

Test statistic (Standard Normal pdf):

$$z = \frac{\hat{p}_m - \hat{p}_f}{\hat{\sigma}_{\hat{p}_m - \hat{p}_f}}$$

where

$$\bar{p} = \frac{x_n + x_f}{n_m + n_f} = \frac{71}{355} = 0.200; \;\; \hat{\sigma}_{\hat{p}_f - \hat{p}_m} = \sqrt{\frac{\bar{p}(1-\bar{p})}{n_f} + \frac{\bar{p}(1-\bar{p})}{n_m}} = 0.042$$

Hence $z = (0.160 - 0.239)/0.042 = -1.667$

Rejection region: $z > z_{0.99} = 2.326$

Since $z > z_{0.99} = 1.645$, then $\underline{H_0 \text{ is not REJECTED}}$.

p-value = `pnorm(-1.667) = 0.0477572`

R solution

```
> prop.test(x=c(43, 28),n=c(180,175),alternative = "greater", correct = TRUE)
```

2-sample test for equality of proportions with continuity correction

data: c(43, 28) out of c(180, 175)

X-squared = 2.9759, df = 1, p-value = 0.04226

```
> prop.test(x=c(43, 28),n=c(180,175),alternative = "greater",correct = FALSE)
```

2-sample test for equality of proportions with continuity correction

data: c(28, 43) out of c(175, 180)

X-squared = 3.4514, df = 1, p-value = 0.0316

Since $p - value = 0.0316 > 0.01$, then $\underline{H_0 \text{ is NOT REJECTED}}$.