Name: SOLUTION

MATH 4377/6308 - Advanced linear algebra I - Summer 2024

Quiz 1

(1) [6 Pts] Let $f : [0, 2\pi] \to [-1, 1]$ be defined by $f(x) = \cos(x)$.

a) Is f one-to-one? Is f onto?

b) Find an interval S, such that $f|_S$ is both one-to-one and onto. Sketch the function on S.

SOLUTION:
(a) f is not 1-1 since $f(0) = f(\pi)$. f is onto.
(b) f one-to-one and onto in the interval $[0, \pi]$ where it is monotonically decreasing.

(2) [4 Pts] Let $x, y \in \mathbb{Z}$. Let $x \sim y$ if and only if $y + 4x$ is an integer multiple of 5. Prove that $\sim$ is a transitive relation.

SOLUTION:
If $y + 4x = 5m$ and if $z + 4y = 5n$, then (using these two equations to express z and $4x$) $z + 4x = (5n - 4y) + (5m - y) = 5(n + m) - 5y = 5(n + m - y)$, which is a multiple of 5.