Name: SOLUTION

MATH 4377/6308 - Advanced linear algebra I - Summer 2024

Quiz 2

(1) [6 Pts] Determine if the following subsets of the vector space of 2×2 matrices with real entries are subspaces. You must justify your answer.

a) $\left\{ \begin{bmatrix} a & b \\ c & -a \end{bmatrix} : a, b, c \in \mathbb{R} \right\}$

b) $\left\{ \begin{bmatrix} a & ab \\ ab & b \end{bmatrix} : a, b \in \mathbb{R} \right\}$

SOLUTION.

(a) This is a subspace. (i) For any $\alpha \in \mathbb{R}$, $\begin{bmatrix} a & b \\ c & -a \end{bmatrix} = \begin{bmatrix} \alpha a & \alpha b \\ \alpha c & -\alpha a \end{bmatrix} = \begin{bmatrix} \tilde{a} & \tilde{b} \\ \tilde{c} & -\tilde{a} \end{bmatrix}$, for $\tilde{a}, \tilde{b}, \tilde{c} \in \mathbb{R}$.

(ii) $\begin{bmatrix} a & b \\ c & -a \end{bmatrix} + \begin{bmatrix} a' & b' \\ c' & -a' \end{bmatrix} = \begin{bmatrix} a+a' & b+b' \\ c+c' & -(a+a') \end{bmatrix} = \begin{bmatrix} \tilde{a} & \tilde{b} \\ \tilde{c} & -\tilde{a} \end{bmatrix}$, for $\tilde{a}, \tilde{b}, \tilde{c} \in \mathbb{R}$.

(b) This is not a subspace. Note that:

$$\begin{bmatrix} a & ab \\ ab & b \end{bmatrix} + \begin{bmatrix} a' & a'b' \\ a'b' & b' \end{bmatrix} = \begin{bmatrix} a+a' & ab+a'b' \\ ab+a'b' & b+b' \end{bmatrix} \neq \begin{bmatrix} a+a' & (a+a')(b+b') \\ (a+a')(b+b') & b+b' \end{bmatrix}$$

(2) [4 Pts] Mark each statement True or False. If True, justify your answer, if False, show a counter-example.

a) A subset of a linearly dependent sets are is linearly dependent.

b) A subset of a linearly independent sets are is linearly independent.

SOLUTION.

(a) FALSE. Let $S = \{(1,0), (2,0)\} \subset \mathbb{R}^2$. S is l.d., but $S_1 = \{(1,0)\} \subset S$ is l.i.

(b) TRUE. If $S \in V$ is a l.i. subset of a vector space V and subset of it is l.i. since no vector is in the linear combination of the other vectors.