Name: SOLUTION

MATH 4377/6308 - Advanced linear algebra I - Summer 2024

Quiz 5

Exercises:

(1) Mark each statement True or False. Justify each answer. (If true, cite appropriate facts or theorems. If false, explain why or give a counterexample that shows why the statement is not true in every case).

a) If B is a matrix that can be obtained by performing an elementary row operation on a matrix A, then A can be obtained by performing an elementary row operation on B.

b) The rank of a matrix is equal to the number of its nonzero columns.

(a) True. Since $B = EA$ with E an elementary matrix, it follows that $A = E^{-1}B$ where the inverse V is also an elementary matrix.

(b) False. For example, the rank of $A = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}$ is 1, which is not equal to the number of its nonzero columns.

(2) Determine the values of the parameter k such that the following system of equations has unique solution, no solution or infinitely many solutions.

$$\begin{aligned} x + z &= k \\ kx + 2y &= 1 \\ -3x + y &= -k \end{aligned}$$

Row reduce: :

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & k \\ k & 2 & 0 & 1 \\ -3 & 1 & 0 & -k \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & 1 & k \\ 0 & 2 & -k & 1-k^2 \\ 0 & 1 & 3 & 2k \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & 1 & k \\ 0 & 1 & 3 & 2k \\ 0 & 2 & -k & 1-k^2 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & 1 & k \\ 0 & 1 & 3 & 2k \\ 0 & 0 & -k-6 & 1-k^2-4k \end{array}\right)$$

This shows that the system has no solution if $k = -6$. If $k \neq -6$, the system has unique solution.

(3) Consider the following matrices and determine if they are invertible or not. Justify your answer.

(a) $\begin{pmatrix} 0 & -1 & -1 \\ 1 & 3 & 2 \\ 1 & 0 & -1 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & -1 & 0 \\ 3 & 0 & 0 \\ 0 & 0 & -2 \end{pmatrix}$

(a) NOT INVERTIBLE since the second row is linearly dependent with the other rows, namely $r2 = r3 - 2r1$.

(b) INVERTIBLE since the matrix can be transformed into a diagonal matrix (with non-zero diagonal entries) by changing the order of row 2 and row 1.