

Name:

MATH 4377/6308 - Advanced linear algebra I - Summer 2024

Quiz 6

(1) Prove that if $A, B \in M^{n,n}(F)$ are similar, then $\det(A) = \det(B)$.

(2) Mark each statement True or False. If true, cite appropriate facts or theorems. If false, give a counterexample that shows why the statement is not true.

- a) Every 2×2 matrix has 2 distinct eigenvalues.
- b) The sum of two eigenvalues of a matrix A is also an eigenvalue of A .
- c) The sum of two eigenvectors of a matrix A is always an eigenvector of A .
- d) Two distinct eigenvectors corresponding to the same eigenvalue are always linearly dependent.

(3) The matrix $A = \begin{pmatrix} -1 & 2 \\ 3 & -2 \end{pmatrix}$ has eigenvalues $\lambda_1 = -4, \lambda_2 = 1$. Find a matrix Q such that

$$Q^{-1}AQ = \begin{pmatrix} -4 & 0 \\ 0 & 1 \end{pmatrix}$$