(a) $\begin{cases} x - y = k-1 \\ x+2y = 0 \end{cases}$

$$\left(\begin{array}{cc|c} 1 & -1 & k-1 \\ 1 & 2 & 0 \end{array}\right) \to \left(\begin{array}{cc|c} 1 & -1 & k-1 \\ 0 & 3 & 1-k \end{array}\right)$$

$\text{rank}(A) = \text{rank}(A|b)$

system has unique solution for any $k \in \mathbb{R}$

(b) $\begin{cases} kx - y = k-1 \\ kx + 2y = 1 \end{cases}$

$$\left(\begin{array}{cc|c} k & -1 & k-1 \\ k & 2 & 1 \end{array}\right) \to \left(\begin{array}{cc|c} k & -1 & k-1 \\ 0 & 3 & k \end{array}\right)$$

If $k=0$, NO SOLUTION ($\text{rank}(A) \neq \text{rank}(A|b)$)

If $k \neq 0$, unique solution ($\text{rank}(A) = \text{rank}(A|b)$)

(c) $\begin{cases} -x + y = k+2 \\ kx - ky = 1 \end{cases}$

$$\left(\begin{array}{cc|c} -1 & 1 & k+2 \\ k & -k & 1 \end{array}\right) \to \left(\begin{array}{cc|c} 1 & -1 & k+2 \\ 0 & 0 & k^2+2k+1 \end{array}\right)$$

If $k \neq 0$

If $k^2+2k+1 = 0$, i.e. $k = -1$, system has infinitely many solutions

If $k \neq -1$, system has no solutions

(c) $\begin{cases} x-y = k-2 \\ kx-ky = -1 \end{cases}$

$$\left(\begin{array}{cc|c} 1 & -1 & k-2 \\ k & -k & -1 \end{array}\right) \to \left(\begin{array}{cc|c} 1 & -1 & k-2 \\ 0 & 0 & -k^2+2k-1 \end{array}\right)$$

$$\downarrow$$

$$k^2-2k+1 = (k-1)^2$$

If $k=1$, the system is consistent and has INFINITELY MANY SOLUTIONS.

If $k \neq 1$, the system has NO solution

(d)

$$\left(\begin{array}{ccc|c} 0 & 1 & 1 & k-1 \\ k & 2 & 0 & 1 \\ -3 & 1 & 0 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & \frac{1}{3} & 0 & 0 \\ k & 2 & 0 & 1 \\ 0 & 1 & 1 & k-1 \end{array}\right) \to \left(\begin{array}{ccc|c} -1 & \frac{1}{3} & 0 & 0 \\ 0 & 2+\frac{k}{3} & 0 & 1 \\ 0 & 1 & 1 & k-1 \end{array}\right)$$

$$\to \left(\begin{array}{ccc|c} -1 & \frac{1}{3} & 0 & 0 \\ 0 & 2+\frac{k}{3} & 0 & 1 \\ 0 & 0 & -\left(2+\frac{k}{3}\right) & (k-k)\left(2+\frac{k}{3}\right)+1 \end{array}\right)$$

NO SOLUTION if $2+\frac{k}{3}=0 \iff k=-6$

UNIQUE SOLUTION if $k \neq -6$

(e)

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & k-1 \\ 1 & 1 & 1 & 1 \\ -3 & 1 & 0 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 1 & k-1 \\ 0 & 0 & 0 & k-2 \\ 0 & 4 & 3 & 3k-3 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 1 & k-1 \\ 0 & 4 & 3 & 3k-3 \\ 0 & 0 & 0 & k-2 \end{array}\right)$$

If $k \neq 2$, NO SOLUTIONS

If $k=2$, INFINITELY MANY SOLUTIONS