Name:

MATH 4377/6308 - Advanced linear algebra I - Summer 2024
Homework 1

Exercises:

1. Let $A = \{1, 2, 5\}$, $B = \{4, 5\}$, $C = \{4, 6\}$. Explicitly write down the sets:

$$A \cup B, \ A \cap (B \cup C), \ B \cap (A \backslash B), A \times C.$$

2. Let $x, y \in \mathbb{Z}$. Prove if the following relations are equivalence relations or not:

   a) $x \sim y$ if and only if $x - y < 10$.

   b) $x \sim y$ if and only if $x \cdot y \geq 0$.

   c) $x \sim y$ if and only if $x - y$ is even.

3. Give an example of a set A and a relation on A which is reflexive and transitive but not symmetric.

4. Let $f : \{0, 1, 2, 3, 4\} \to \mathbb{N}$, $n \to n^3 + n$.

   a) Find domain, codomain, and range of f.

   b) Is f one-to-one?

   c) Is f onto?

5. Let $f : [0, 2\pi] \to [-1, 1]$ be defined by $f(x) = \sin(x)$.

   a) Is f one-to-one? Is f onto?

   b) Find an interval S, such that $f|_S$ is both one-to-one and onto.

6. Let $z = 1 + i2$, $w = 1 - i3$. Write: $\bar{z}$, $z + w$, zw, $\dfrac{1}{w}$ in the form $a + ib$. Finally write $|z|$.

7. Let $x, y \in \mathbb{Z}$. Let $x \sim y$ if and only if $y + 4x$ is an integer multiple of 5. Prove that $\sim$ is an equivalence relation.