Name:

MATH 4377/6308 - Advanced linear algebra I - Summer 2024

Homework 2

Exercises:

1. Mark each statement True or False. Justify each answer.
   a) A subset H of a vector space V is a subspace of V if the zero vector is in H.
   b) A subspace is also a vector space.
   c) If u is a vector in a vector space V , then $(-1)u$ is the same as the negative of u.
   d) A vector space is also a subspace.
   e) $\mathbb{R}^2$ is a subspace of $\mathbb{R}^3$.
   f) If f is a function in the vector space V of all real-valued functions on $\mathbb{R}$ and if $f(t) = 0$ for some t, then f is the zero vector in V .
   g) If S is a linearly dependent set, then each vector in S is a linear combination of other vectors in S.
   h) Any set containing the zero vector is linearly dependent.
   i) Subsets of linearly dependent sets are linearly dependent.

2. (2 points) Determine if the following subsets of $\mathbb{R}^3$ are subspaces:
   a) $\{(a, b, c) \in \mathbb{R}^3 : 2a - 3c = 0\}$
   b) $\{(a, b, c) \in \mathbb{R}^3 : a - 2b + c = 1\}$
   c) $\{(a, b, c) \in \mathbb{R}^3 : 2a = c\}$
   d) $\{(a, b, c) \in \mathbb{R}^3 : 2a = 5c \text{ and } 4b = a + c\}$

3. Determine if the following subsets of the vector space of 2×2 matrices with real entries are subspaces:
   a) $\left\{ \begin{bmatrix} a & b \\ c & 0 \end{bmatrix} : a, b, c \in \mathbb{R} \right\}$
   b) $\left\{ \begin{bmatrix} a & b^2 \\ b & a^2 \end{bmatrix} : a, b \in \mathbb{R} \right\}$

4. A real-valued function f defined on the real line is called an *even function* if $f(t) = f(-t)$ for each real number t. Prove that the set of even functions is a subspace with the usual addition and scalar multiplication for functions. (You may assume as true that the set of real-valued functions f defined on the real line is a vector space with the usual addition and scalar multiplication for functions.)

5. Suppose $u_1, \ldots, u_p$ and $v_1, \ldots, v_p$ are vectors in a vector space V, and let

$$H = span(u_1, \ldots, u_p), \quad K = span(v_1, \ldots, v_p)$$

   Prove that $H + K = span(u_1, \ldots, u_p, v_1, \ldots, v_p)$