Name:

MATH 4377/6308 - Advanced linear algebra I - Summer 2024

Homework 3

Exercises:

(1) Decide if each of the following statements is True or False. Justify each answer. If true, cite appropriate facts or theorems. If false, explain why or give a counterexample that shows why the statement is not true in every case.

Let $v_1, \ldots, v_p$ be vectors in a non-zero finite-dimensional vector space V, and let $S = \{v_1, \ldots, v_p\}$.

a) The set of all linear combinations of $v_1, \ldots, v_p$ is a vector space.

b) If $\{v_1, \ldots, v_{p-1}\}$ spans V, then S spans V.

c) If $\{v_1, \ldots, v_{p-1}\}$ is linearly independent, then so is S.

d) If S is linearly independent, then S is a basis for V.

e) If S is linearly independent, then $dimV = p$.

f) If $V = spanS$, then some subset of S is a basis for V.

g) If $V = spanS$, then $dimV = p$.

h) If $dimV = p$ and $V = spanS$, then S cannot be linearly dependent.

i) A plane in $\mathbb{R}^3$ is a two-dimensional subspace.

(2) The vectors $u_1 = (1, 1, 1, 1)$, $u_2 = (0, 1, 1, 1)$, $u_3 = (0, 0, 1, 1)$, and $u_4 = (0, 0, 0, 1)$ form a basis for $\mathbb{R}^4$. Find a unique representation of an arbitrary vector $(a, b, c, d) \in \mathbb{R}^4$ as a linear combination of u_1, u_2, u_3, and u_4.

(3) Let $L = \{(1, 2, 1, 3), (0, 0, 1, 1)\}$. Let $G = \{(1, 2, -2, 0), (1, 0, 0 - 1), (0, 1, 1, 1), (1, 2, 2, 4)\}$. You can assume without proof that G spans $\mathbb{R}^4$. Find a subset $H \subset G$ such that $H \cup L$ spans $\mathbb{R}^4$. You need to justify that the set you build spans $\mathbb{R}^4$.

(4) Let $T : \mathbb{R}^5 \to \mathbb{R}^3$ be given by

$$T(a_1, a_2, a_3, a_4, a_5) = (a_1 + 2a_2 - a_3, -a_2 + 3a_3, -a_1 - a_2 - 2a_3)$$

(a) Verify that T is linear.

(b) Find bases for the null space and the range of T.