Name:

MATH 4377/6308 - Advanced linear algebra I - Summer 2024

Homework 4

Exercises:

(1) Consider the linear transformation:

$$T : P_2(\mathbb{R}) \to P_3(\mathbb{R}), T(p(x)) = 2p'(x) + \int_0^x p(t)dt$$

Prove that T is one-to-one but not onto.

(2) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be given by

$$T(a_1, a_2) = (a_1 + a_2, a_1 - a_2).$$

(a) Write $[T]_\beta^\gamma$ with $\beta = \{(1,0),(0,1)\}$ and $\gamma = \{(1,0),(0,1)\}$.

(b) Write $[T]_\beta^{\tilde{\gamma}}$ with $\beta = \{(1,0),(0,1)\}$ and $\tilde{\gamma} = \{(1,2),(1,1)\}$.

(3) Let $T : P_1(\mathbb{R}) \to P_1(\mathbb{R})$ and $U : P_1(\mathbb{R}) \to \mathbb{R}^2$ be the linear transformations defined by

$$T(p(x)) = p'(x) + 2p(x), \quad U(a + bx) = (a + b, a)$$

Let β and γ be the standard ordered bases of $P_1(\mathbb{R})$ and $\mathbb{R}^2$, respectively. Find $[T]_\beta$, $[U]_\beta^\gamma$ and $[U \circ T]_\beta^\gamma$.

(4) For the following pairs of vector spaces V and W, define an explicit isomorphism or explain why no isomorphism exists between such spaces.

(a) $V = \mathbb{R}^2$, $W = M^{1,1}$

(b) $V = \mathbb{R}^4$, $W = M^{2,2}$

(c) $V = \mathbb{R}^4$, $W = P_4(\mathbb{R})$

(d) $V = \mathbb{R}^4$, $W = P_3(\mathbb{R})$

(e) $V = \mathbb{R}^2$, $W = \mathbb{C}$ (space of complex numbers)

(5) Let $\beta' = \{(3,1),(2,4)\}$, $\beta = \{(1,1),(1,-1)\}$. Find the change of coordinates matrix $Q = [I_V]_{\beta'}^{\beta}$.