Name:

MATH 4377/6308 - Advanced linear algebra I - Summer 2024

Homework 5

Exercises:

(1) Mark each statement True or False. Justify each answer. (If true, cite appropriate facts or theorems. If false, explain why or give a counterexample that shows why the statement is not true in every case).

a) If B is a matrix that can be obtained by performing an elementary row operation on a matrix A, then A can be obtained by performing an elementary row operation on B.

b) The rank of a matrix is equal to the number of its nonzero columns.

c) The product of two matrices always has rank equal to the lesser of the ranks of the two matrices.

d) Elementary row operations preserve rank.

e) The rank of a matrix is equal to the maximum number of linearly independent rows in the matrix.

f) An $n \times n$ matrix having rank n is invertible.

g) Any homogeneous system of linear equations has at least one solution.

h) If the homogeneous system corresponding to a given system of linear equations has a solution, then the given system has a solution.

i) The solution set of any system of m linear equations in n unknowns is a subspace of F_n.

j) If A is an $n \times n$ matrix with rank n, then the reduced row echelon form of A is I_n.

(2) Apply the method of Gaussian elimination to find the solution set of the following system of equations or show that it has no solution. Write the elementary matrices that you apply to transform the matrix of coefficients.

$$\begin{aligned} 3x + 3y + z &= 15 \\ 2x - y + z &= 3 \\ x + 4y &= 9 \end{aligned}$$

(3) Determine the values of the parameter k such that the following system of equations has unique solution, no solution or infinitely many solutions.

$$\begin{aligned} ky + kz &= k \\ kx + 2y &= 1 \\ -3x + y &= -k \end{aligned}$$

(4) Determine the values of the parameter k such that the following system of equations has unique solution, no solution or infinitely many solutions.

$$\begin{aligned} 2x + y - 5z &= k \\ -x + 3y + kz &= 0 \\ 3x + 2y - 9z &= 0 \\ -x + y - 2z &= 0 \end{aligned}$$

(5) Consider the following matrices and find the inverse, if it exists. If it does not exist, explain why.

(a) $\begin{pmatrix} 0 & 1 & -1 \\ 1 & 2 & 0 \\ 0 & 2 & -2 \end{pmatrix}$, (b) $\begin{pmatrix} 0 & 1 & -1 \\ 1 & 2 & 0 \\ 1 & 3 & -1 \end{pmatrix}$ (c) $\begin{pmatrix} 0 & 1 & -1 \\ 1 & 2 & 0 \end{pmatrix}$ (d) $\begin{pmatrix} 0 & 2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -2 \end{pmatrix}$

(6) For each of the following matrices, determine the values of the parameter k such that the matrix is invertible.

(a) $\begin{pmatrix} 2 & k & -1 \\ 0 & 1 & -1 \\ 1 & 2 & 0 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & k & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 3 \end{pmatrix}$ (c) $\begin{pmatrix} 2 & k & -1 \\ 1 & 1 & -1 \\ 3 & 3 & -3 \end{pmatrix}$