Additions and corrections for

Variational Principles for Finite Dimensional Initial Value Problems.

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Abstract. The following notes provide some corrections and material to complete the proof of theorem 6.2. of the paper which appeared in Contemporary Mathematics, **426,** pp 45-56.

Equation (2.1) should read.

$$0 \in \dot{u}(t) + \partial_u V(t, u(t)) \qquad \text{for all } t \in [0,T].$$

The material around Equation (6.6) should read

$\mathcal{L} : [0,T] \times \mathbb{R}^m \times \mathbb{R}^m \to \mathbb{R}$ is defined by

$$\mathcal{L}(t,u,z) := q^*(t, z + B(t)u - f(t)) + q(t,u) - \langle f(t), u \rangle.$$

The second paragraph of theorem 6.2 was a stub whose content should be replaced by the following result which proves the coercivity of J on a closed convex subset of $H^1((0,T);\mathbb{R}^m)$.

Lemma 6.3. *Assume (A1) holds and J is defined by (6.5)-(6.6), then J is coercive on K_0.*

Proof. Write the Lagrangian (6.7) as

$$2\,\mathcal{L}(t,u,z) = l_2(t,u,z) + l_2(t,u,z) + l_0(t)$$

where l_2 is the quadratic part

$$l_2(t,u,z) := \langle A_S(t)u, u \rangle + \langle A_S(t)^{-1}(Bu + z), Bu + z \rangle$$

of $\mathcal{L}$, while l_1 includes the terms that are linear in u or z and $l_0(t) := \langle A_S(t)^{-1} f(t), f(t) \rangle$.

Introduce the family of inner products

$$[x,y]_t := \langle A_S(t)^{-1}x, y \rangle \quad \text{for} \quad t \in [0,T].$$

For each t, (6.4) shows this is an equivalent inner product to the usual inner product on $\mathbb{R}^m$. The standard inequality

$$2[z, Bu]_t \;\leq\; \epsilon \, \|z\|_t^2 \;+\; \epsilon^{-1}\|Bu\|_t^2 \quad \text{for all} \quad z, u \in \mathbb{R}^m$$

holds for any $\epsilon > 0$. Thus the expression l_2 obeys

$$2\, l_2(t, u, z) \;\geq\; a_0\|u\|^2 \;+\; c\|z\|_t^2 - c\epsilon^{-1}\|Bu\|_t^2 \tag{0.0.1}$$

where $c = 1 - \epsilon$. From (A0) and (A1), since the matrices are uniformly bounded on [0,T], there is a constant $b > 0$ such that

$$\|Bx\|_t \;\leq\; b \, \|x\|_2 \quad \text{for all} \quad t \in [0, T], \;\, x \in \mathbb{R}^m.$$

Choose ϵ so that

$$\frac{b^2}{a_0 + b^2} < \epsilon < 1.$$

Substitute in (0.0.1), then

$$2\, l_2(t, u(t), \dot{u}(t)) \;\geq\; c \, \|\dot{u}(t)\|_t^2 \;+\; c_1 \, \|u(t)\|_2^2 \tag{0.0.2}$$

where $c > 0$. The other terms in the Lagrangian (6.7) grow at most linearly in $u, \dot{u}$, so this implies that the functional J defined by (6.5) is coercive on K_0.

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