July 7, 2014

Name:

Final Math 3330

You have **120** minutes to complete the test. Each problem is worth **20** points. You cannot use any books or notes.

1. Label each of the following statements as either true of false.
   (a) The integers $\mathbb{Z}_n$ form a commutative ring.
   (b) If $\mathbb{D}$ is an integral domain then every homomorphic image of $\mathbb{D}$ is an integral domain.
   (c) A subring of a domain is a domain.
   (d) The ring $\mathbb{Z} \times \mathbb{Z}$ with coordinate wise definition of addition and multiplication is a domain.
   (e) A finite integral domain is a field.
   (f) For no n the ring $\mathbb{Z}_n$ can be a field.
   (g) A subring of a field is a field.
   (h) A subring of a field is an integral domain.
   (i) $\mathbb{Z}_2$ is a field.
   (j) $\mathbb{Z}_2 \times \mathbb{Z}_2$ is a field of four elements.
2. Consider the set $R = \{[0], [3], [6], [9]\} \subseteq \mathbb{Z}_{12}$ with addition and multiplication as defined in $\mathbb{Z}_{12}$. Prove or disprove that R is an ideal.
3. (a) Explain why [36] cannot have a multiplicative inverse in $\mathbb{Z}_{38}$.
   (b) Find the multiplicative inverse of [35] in $\mathbb{Z}_{38}$.
4. Let H be a normal subgroup of the group G. If the order of the quotient group G/H is m, prove that $g^m \in H$ for all $g \in G$.
5. Prove that every homomorphism from a field $\mathbb{F}_1$ to a field $\mathbb{F}_2$ is injective.
6. (a) Define that $\mathbb{A} = (A, +, -, 0, \cdot, 1)$ is a ring with unit.
   (b) State the definition of a closed subset of a ring.
   (c) State the definition of an ideal of a ring.
7. Prove that the set of all complex numbers of the form $m + ni$, where $m \in \mathbb{Z}$ and $n \in \mathbb{Z}$ is a subring of $\mathbb{C}$. This ring is called the *Gaussian integers*. What is (1+2i)(1-2i)?
8. Assume that the ideal I of a ring A contains an invertible element. Prove that $I = A$.
9. Let R_1 and R_2 be subrings of the ring A. Prove that $R_1 \cap R_2$ is a subring of A.
10. The polynomials with real coefficients form a ring $\mathbb{R}[x]$. Find all invertible elements of $\mathbb{R}[x]$.