

Name:

Test 1 Math 5330

You have **60** minutes to complete the test. Each problem is worth **20** points. You cannot use any books or notes.

1. label each of the following statements as either true or false.
 - (a) $A \cup \emptyset = A$ is true for any set A .
 - (b) If $A \cup X = A$ holds for every set A then X must be \emptyset .
 - (c) The set $\{\emptyset\}$ has only \emptyset as a subset.
 - (d) Let $f : A \rightarrow B$ be any map. Then $f(f^{-1}(T)) = T$ is true for every subset T of B .
 - (e) Let $f : A \rightarrow A$ be an injective map on a finite set. Then f is surjective.
 - (f) Assume that $f : A \rightarrow B$ is injective. Then there is a unique map $g : B \rightarrow A$ such that $g \circ f = id_A$.
 - (g) Let E be an equivalence relation on a set A . Then any two equivalence classes of E contain the same number of elements.
 - (h) Let π_1 and π_2 be the partitions of two equivalence relations E_1 and E_2 on a set A . Then if $\pi_1 = \pi_2$ one has that $E_1 = E_2$.
 - (i) If the map $f : A \rightarrow B$ is injective then the equivalence kernel for f , \sim_f is the equality relation on A .
 - (j) The equivalence kernel of a function $f : A \rightarrow \{c\}$ is $A \times A$.
2. Let $f : A \rightarrow B$ and $g : B \rightarrow C$ be functions. Assume that $g \circ f$ is injective. Prove that f must be injective.
3. For the map $f : \mathbb{N} \rightarrow \mathbb{N}, n \mapsto 2n$ on the set of natural numbers find a map $g : \mathbb{N} \rightarrow \mathbb{N}$ such that $g \circ f = id_{\mathbb{N}}$ and prove that there is no map h such that $f \circ h = id_{\mathbb{N}}$.
4. In each of the following parts, a relation R is defined on the set \mathbb{R} of all real numbers. Determine in each case whether R is an equivalence relation.
 - (a) xRy iff $x = -y$
 - (b) xRy iff $xy \geq 0$
 - (c) xRy iff $\sin(x) = \sin(y)$
 - (d) xRy iff $|x - y| \leq 1$.
5. Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{a, b, c\}$ and f the map $1 \mapsto a, 2 \mapsto c, 3 \mapsto b, 4 \mapsto c, 5 \mapsto a$. Find the partition of the equivalence kernel for f .