

Practice sheet for Test 3, Math 3330, Spring 2016

1. Label each of the following statement as either true or false/

- (a) Every cyclic group is abelian.
- (b) If a subgroup H of a group G is cyclic, then G must be cyclic.
- (c) If there exists a positive integer m such that $a^m = e$, where a is an element of a group G then the order of a is m .
- (d) Two cyclic groups of the same order are isomorphic.
- (e) Two abelian groups of the same order are isomorphic.
- (f) If G is a finite group of order n then the order m of an element a of G divides n .
- (g) If H is a normal subgroup of G then every coset xH is a subgroup of G .
- (h) If G is abelian and N a normal subgroup of G then G/N is abelian.
- (i) Let H be any subgroup of a group G . Then $aH = Ha$ implies that $ah = ha$ for all a in H .
- (j) If all subgroups of a group G are normal then G must be abelian.

2. Prove that any group G of order p where p is a prime number must be cyclic.

3. Let G be any group. The center $Z(G)$ is defined as $Z(G) = \{a | ax = xa \text{ for all } x \in G\}$. Prove that $Z(G)$ is a normal subgroup of G .

4. Let H be a subgroup of G of index 2. Prove that H is normal.

5. Let G be a group and let H and K be subgroups of G such that $|H| = 12$ and $|K| = 5$. Prove that $H \cap K = \{e\}$.

6. Is $(\mathbb{Z}_{14}, +)$ isomorphic to a subgroup of $(\mathbb{Z}_{56}, +)$? You must explain your answer