# MATH 4332

# Homework 1
# due Monday, January 23rd

## NAME: .....

Q1. Find the pointwise limits[1] of the following sequences of functions on the specified domain. In each case describe the continuity properties of the limit function.

(a) $f_n(x) = x^n$, $x \in [0, 1]$.

(b) $f_n(x) = \tan^{-1}(nx)$, $x \geq 0$.

(c) $f_n(x) = \frac{nx}{1+n^2x^2}$, $x \in \mathbb{R}$.

Is the convergence for any of these sequences uniform? Why/Why not?

Q2. Show that if $f_n(x) = x^{n-1}(1 - 2x^n)$ then

(a) $\sum_{n=1}^{\infty} \int_0^1 f_n(x)\, dx = 0$.

(b) $\sum_{n=1}^{\infty} f_n(x) = \frac{1}{1+x}$.

and so deduce that it is not possible to interchange order of integration and summation.

Q3. Show that $\sum_{n=1}^{\infty} \frac{1}{(n+x)^2}$ converges uniformly on $x \geq 0$ but that the sum cannot be integrated from 0 to $\infty$. (You may assume that *if* the integral exists then it can be obtained by term-by-term integration.)

Q4. Find two simple examples of convergent infinite series of functions which we cannot differentiate term-by-term. (IE the sum of the derivatives is either divergent at some points or does not converge to the derivative of the infinite sum.)

**Guidelines for homework to be marked**
1. Use lined paper and *write neatly along lines moving monotonically from top to bottom of the page.*
2. **Staple sheets together**.
3. Do not fold.
4. No more than 5% of work should be crossed out.

Homework will count 25% of total mark for course (same rules as last semester).

[1]The pointwise limit $f$ of $(f_n)$ is defined by $f(x) = \lim_{n\to\infty} f_n(x)$, for all $x$ in the domain of definition of $f_n$ — assuming limit exists.