Department of Mathematics

University of Houston

Scientific Computing Seminar

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Divergence-conforming B-spline Discretizations for Viscous Incompressible Flows

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3:00 PM- 4:00 PM
Room 646 PGH

Abstract:

The Navier-Stokes equations are infused with vast geometric structure, evidenced by a wide array of balance laws for momentum, angular momentum, circulation, energy, vorticity, enstrophy, and helicity. These balance laws are considered to be of prime importance in the evolution of laminar and turbulent flow structures, and they are even believed to play a role in the regularity of Navier-Stokes solutions. The key to unlocking much of the geometric structure of Navier-Stokes flow is precisely its volume-preserving nature, yet most numerical methods only satisfy the incompressibility constraint in an approximate sense. Consequently, such methods do not obey many fundamental laws of physics. In particular, advective and conservative semi-discrete approximations of the Navier-Stokes equations are typically guaranteed to balance energy only if incompressibility is satisfied pointwise. This is especially concerning as energy plays a fundamental role in numerical stability.

In this talk, I will discuss recently developed B-spline discretizations for viscous incompressible flows which satisfy the incompressibility constraint pointwise. As incompressibility is satisfied pointwise, these discretizations replicate the geometric structure of the Navier-Stokes equations and properly balance energy, enstrophy, and helicity. Furthermore, these discretizations converge to suitable weak solutions of the Navier-Stokes equations which satisfy a local energy balance in space-time. These attributes in conjunction with the resolution properties of B-splines make these discretizations an attractive candidate for reliable numerical simulation of viscous incompressible flows. In this talk, I will specifically address the mathematical theory underlying divergence-conforming B-spline discretizations of the Navier-Stokes equations and the development of a variational multiscale subgrid-scale model specifically tailored for such discretizations. I will also present numerical results illustrating the promise of this new technology.