Department of Mathematics

University of Houston

Scientific Computing Seminar

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Numerical Solution of Second and Fourth Order Total Variation Flow Problems

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Abstract: Second and fourth order Total Variation Flow (TVF) problems are L^2- and H^{-1}-gradient flows associated with an energy functional in terms of the total variation. They give rise to nonlinear second and fourth order parabolic PDEs representing very singular diffusion processes and have applications in image processing and materials science. As specific examples, we consider the Kobayashi-Warren-Carter (KWC) model describing the crystallization of single crystals and surface relaxation below the roughening temperature. The KWC model represents a two phase field approach where the free energy is given by two field variables, the orientation angle and the local degree of crystallinity. The associated L^2-gradient flows are given by a coupled system of two nonlinear second order parabolic PDEs. Based on a regularization of the weighted total variation in the orientation angle and a discretization in time by the backward Euler scheme, we suggest an energy stable splitting scheme that requires the successive solution of two nonlinear elliptic problems. The discretization in space is taken care of by standard Lagrangian finite elements with respect to a simplicial triangulation of the computational domain. In case of surface relaxation below the roughening temperature we consider the same regularization of the energy functional and a scaling of the associated fourth order problem both in space and time. For discretization in time we use again the backward Euler scheme, whereas for discretization in space we choose a C^0 Interior Penalty Discontinuous (C^0 IPDG) method. Numerical results are given illustrating the performance of the splitting scheme for the KWC model and the C^0IPDG method for the surface relaxation.

- This is a joint work with with Chandi Bhandari, Rahul Kumar, and James Winkle.