Answers to Homework 2， Math 4364（Fall 2009）

2.3

1. $f(x) = x^2 - 6$ $\qquad f'(x) = 2x$

$P_0 = 1$

$$P_n = P_{n-1} - \frac{f(P_{n-1})}{f'(P_{n-1})} = P_{n-1} - \frac{P_{n-1}^2 - 6}{2P_{n-1}} \qquad n \geq 1$$

So

$$P_1 = P_0 - \frac{P_0^2 - 6}{2P_0} = 1 - \frac{1-6}{2} = 3.5$$

$$P_2 = P_1 - \frac{P_1^2 - 6}{2P_1} = 3.5 - \frac{3.5^2 - 6}{7} \approx 2.60714$$

3.

a. The Secant method

$P_0 = 3 \qquad P_1 = 2$

$$P_n = P_{n-1} - \frac{f(P_{n-1})(P_{n-1} - P_{n-2})}{f(P_{n-1}) - f(P_{n-2})}, \qquad n \geq 2$$

So

$$P_2 = P_1 - \frac{f(P_1)(P_1 - P_0)}{f(P_1) - f(P_0)} = 2 - \frac{(2^2 - 6)(2-3)}{(2^2 - 6) - (3^2 - 6)} = 2.4$$

$$P_3 = P_2 - \frac{f(P_2)(P_2 - P_1)}{f(P_2) - f(P_1)} = 2.4 - \frac{(2.4^2 - 6)(2.4-2)}{(2.4^2 - 6) - (2^2 - 6)} \approx 2.45455$$

b. The method of False Position

$P_0 = 3 \qquad P_1 = 2$

$f(P_0) = 3 \qquad f(P_1) = -2 \qquad f(P_0)f(P_1) < 0$

$$P_2 = P_1 - \frac{f(P_1)(P_1 - P_0)}{f(P_1) - f(P_0)} = 2 - \frac{(2^2 - 6)(2-3)}{(2^2 - 6) - (3^2 - 6)} = 2.4$$

$f(P_2) = 2.4^2 - 6 = -0.24$ $\qquad\qquad$ $f(P_2)f(P_1) > 0$

So we can choose P_3 as the x-intercept of the line joining $(P_0, f(P_0))$ and $(P_2, f(P_2))$,

$$P_3 = P_2 - \frac{f(P_2)(P_2 - P_0)}{f(P_2) - f(P_0)} = 2.4 - \frac{-0.24(2.4-3)}{-0.24-3} \approx 2.44444$$

c. (b) is closer to $\sqrt{6}$

5c. $P_4 = 0.73909$ with the initial guess $P_0 = 0$

7c. $P_6 = 0.73909$ with the initial guess $P_0 = 0, P_1 = \pi/2$

9c. $P_7 = 0.73908$ with the initial guess $P_0 = 0, P_1 = \pi/2$

16. $P_{15} \approx 1.895488$ with $P_0 = \pi/2$

$P_{19} \approx 1.895489$ with $P_0 = 5\pi$

$P_{156} \approx 1.895499$ with $P_0 = 10\pi$

$f(x) = \frac{1}{2} + \frac{1}{4}x^2 - x\sin x - \frac{1}{2}\cos(2x) = (\frac{1}{2}x - \sin x)^2$ has a zero of multiplicity m with $m > 1$ at the zero $P \approx 1.895488$ (see the graph of f(x) on the following page).

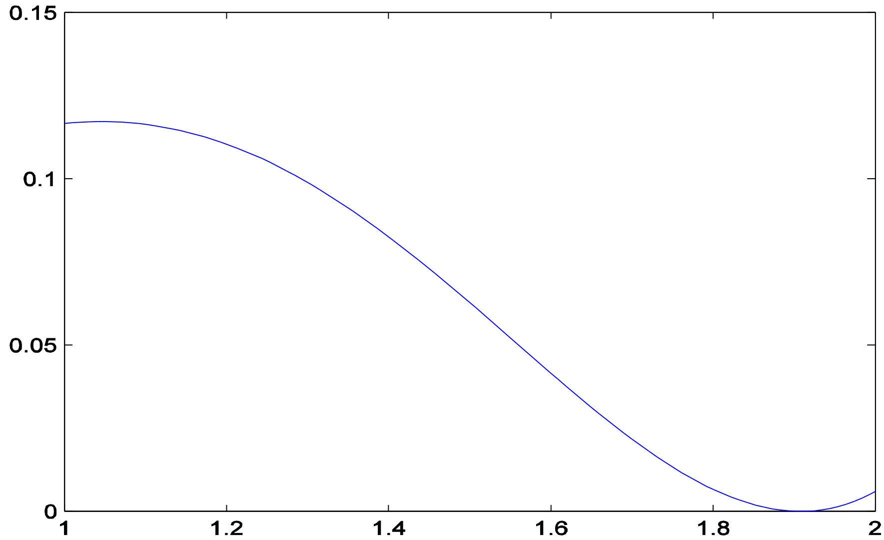

Figure 1: The graph of f(x) on [1 2] shows that at f(x) has a zero of multiplicity m with m > 1 between 1.8 and 2.0.