2.4

1b. $P_{23} \approx -1.4144529$ with $P_0 = -1.5$

$P_{29} \approx -1.4144532$ with $P_0 = -2.0$

Note: You have to use double precision to get better approximate solutions. Also with different codes, your approximate solutions vary slightly.

3b. $P_2 \approx -1.4142416$ with $P_0 = -1.5$

$P_3 = -1.4142136$ with $P_0 = -2.0$

Yes, there is an improvement in speed.

6.

a.

$$\frac{|P_{n+1} - 0|}{|P_n - 0|} = \frac{1/(n+1)}{1/n} = \frac{n}{n+1} \to 1, \qquad as \qquad n \to \infty$$

Therefore, the sequence defined by $P_n = \frac{1}{n}$ converges linearly to $P = 0$

$|P_n - P| \le 5\times10^{-2}$, i.e. $\left|\frac{1}{n} - 0\right| = \frac{1}{n} \le 5\times10^{-2} \Rightarrow n \ge 20$

b.

$$\frac{|P_{n+1} - 0|}{|P_n - 0|} = \frac{1/(n+1)^2}{1/n^2} = \frac{n^2}{(n+1)^2} \to 1, \qquad as \qquad n \to \infty$$

Therefore, the sequence defined by $P_n = \frac{1}{n^2}$ converges linearly to $P = 0$

$|P_n - P| \le 5\times10^{-2}$, i.e. $\left|\frac{1}{n^2} - 0\right| = \frac{1}{n^2} \le 5\times10^{-2} \Rightarrow n \ge \frac{1}{\sqrt{5\times10^{-2}}} \approx 4.47$

So $n \ge 5$

7a.

Given positive integer k

$$\frac{|P_{n+1} - 0|}{|P_n - 0|} = \frac{1/(n+1)^k}{1/n^k} = \left(\frac{n}{n+1}\right)^k \to 1, \qquad as \qquad n \to \infty$$

Therefore, the sequence defined by $P_n = \frac{1}{n^k}$ converges linearly to $P = 0$