

Answers to Homework 3, Math 4363(Fall 2009)

2.5

$$3. \quad g(x) = \cos(x-1), \quad p_0^{(0)} = 2;$$

$$p_1^{(0)} = g(p_0^{(0)}) = \cos(2-1) = 0.54030230;$$

$$p_2^{(0)} = g(p_1^{(0)}) = \cos(0.54030230-1) = 0.89618666$$

$$p_0^{(1)} = p_0^{(0)} - \frac{(p_1^{(0)} - p_0^{(0)})^2}{p_2^{(0)} - 2p_1^{(0)} + p_0^{(0)}} = 0.82642739.$$

6.1

3a.

$$\left(\begin{array}{ccc|c} 4 & -1 & 1 & 8 \\ 2 & 5 & 2 & 3 \\ 1 & 2 & 4 & 11 \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} 4 & -1 & 1 & 8 \\ 0 & 5.5 & 1.5 & -1.0 \\ 0 & 2.3 & 3.8 & 9.0 \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} 4 & -1 & 1 & 8 \\ 0 & 5.5 & 1.5 & -1.0 \\ 0 & 0 & 3.2 & 9.4 \end{array} \right)$$

$$x_3 = \frac{9.4}{3.2} \approx 2.9;$$

$$x_2 = \frac{-1.0 - 1.5x_3}{5.5} \approx \frac{-1.0 - 4.4}{5.5} \approx -0.98;$$

$$x_1 = \frac{8.0 - x_3 + x_2}{4.0} \approx \frac{6.1 - 0.98}{4.0} \approx \frac{4.1}{4.0} \approx 1.0.$$

5a.

$$\left(\begin{array}{ccc|c} 1 & -1 & 3 & 2 \\ 3 & -3 & 1 & -1 \\ 1 & 1 & 0 & 3 \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} 1 & -1 & 3 & 2 \\ 0 & 0 & -8 & -7 \\ 0 & 2 & -3 & 1 \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} 1 & -1 & 3 & 2 \\ 0 & 2 & -3 & 1 \\ 0 & 0 & -8 & -7 \end{array} \right)$$

$$x_3 = \frac{-7}{-8} = 0.875;$$

$$x_2 = \frac{1 + 3x_3}{2} = 1.8125;$$

$$x_1 = 2 - 3x_3 + x_2 = 1.1875.$$

With one row interchange (row2 and row 3) required.

Project 0

(a) Bisection Method with the interval [0,1]

I	P	F(P)
1	5.00000000e-001	-3.7758256e-001
2	7.50000000e-001	1.8311131e-002
3	6.25000000e-001	-1.8596312e-001
4	6.87500000e-001	-8.5334946e-002
5	7.18750000e-001	-3.3879372e-002
6	7.34375000e-001	-7.8747255e-003
7	7.42187500e-001	5.1957117e-003
8	7.38281250e-001	-1.3451498e-003
9	7.40234375e-001	1.9238728e-003
10	7.39257813e-001	2.8900915e-004
11	7.38769531e-001	-5.2815843e-004
12	7.39013672e-001	-1.1959667e-004
13	7.39135742e-001	8.4700731e-005
14	7.39074707e-001	-1.7449347e-005
15	7.39105225e-001	3.3625348e-005
16	7.39089966e-001	8.0879147e-006
17	7.39082336e-001	-4.6807375e-006

Approximate solution $P = 0.73908234$ with $F(P) = -0.00000468$

Number of iterations = 17 Tolerance = $1.00000000e-005$

(b) Fixed-Point Method with $g(x) = \cos x$ and $P_0 = 0.5$

I	P
1	8.77582562e-001
2	6.39012494e-001
3	8.02685101e-001
4	6.94778027e-001
5	7.68195831e-001
6	7.19165446e-001
7	7.52355759e-001
8	7.30081063e-001
9	7.45120341e-001
10	7.35006309e-001
11	7.41826523e-001
12	7.37235725e-001
13	7.40329652e-001
14	7.38246238e-001
15	7.39649963e-001
16	7.38704539e-001
17	7.39341452e-001
18	7.38912449e-001
19	7.39201444e-001
20	7.39006780e-001
21	7.39137911e-001
22	7.39049581e-001
23	7.39109081e-001
24	7.39069001e-001
25	7.39096000e-001
26	7.39077813e-001
27	7.39090064e-001
28	7.39081812e-001

Approximate solution P = 0.73908181

Number of iterations = 28

Tolerance = 1.00000000e-005

(c) Newton's Method with $P_0 = 0.5$

I	P	F(P)
1	7.55222417e-001	2.7103312e-002
2	7.39141666e-001	9.4615381e-005
3	7.39085134e-001	1.1809779e-009
4	7.39085133e-001	0.0000000e+000

Approximate solution = 7.3908513322e-001 with $F(P) = 0.0000000000e+000$

Number of iterations = 4

Tolerance = 1.0000000000e-005

Newton's Method with $P_0 = 1000.$

I	P	F(P)
1	4.52926370e+002	4.5206714e+002
2	1.53858069e+002	1.5485487e+002
3	1.04587393e+001	1.0970158e+001
4	-6.75272122e+001	-6.7510183e+001
5	-3.37696731e+001	-3.3064299e+001
6	7.97891881e+001	8.0105107e+001
7	-1.48434588e+003	-1.4844025e+003
8	9.23828008e+005	9.2382773e+005
9	-2.32850973e+007	-2.3285097e+007
10	1.16486665e+008	1.1648667e+008
11	-6.04529865e+007	-6.0452986e+007
12	-2.45868181e+007	-2.4586818e+007
13	-1.07997047e+007	-1.0799704e+007
14	1.78229217e+007	1.7822921e+007
15	6.19732536e+006	6.1973244e+006
16	-1.75800144e+006	-1.7580007e+006
17	-7.13021129e+005	-7.1302165e+005
18	-3.28397528e+005	-3.2839837e+005
19	3.78232969e+005	3.7823357e+005
20	-1.48250862e+006	-1.4825077e+006
21	-4.63832352e+005	-4.6383259e+005
22	1.55965258e+007	1.5596525e+007
23	7.41640807e+006	7.4164071e+006
24	1.66633926e+005	1.6663468e+005
25	-3.11782940e+005	-3.1178323e+005
26	-1.52580394e+005	-1.5258128e+005
27	-4.80452573e+004	-4.8044621e+004

28	-2.09214056e+004	-2.0921375e+004
29	-1.04582619e+004	-1.0457267e+004
30	1.15918654e+003	1.1601847e+003
31	6.58182866e+001	6.6806274e+001
32	7.95417669e+000	8.0542042e+000
33	3.91695059e+000	4.6311211e+000
34	-1.15186663e+001	-1.2018227e+001
35	-5.07899173e+000	-5.4374377e+000
36	-2.26683976e+000	-1.6256533e+000
37	4.72176302e+000	4.7123891e+000
38	-1.07251002e+005	-1.0725002e+005
39	-1.55611500e+004	-1.5560484e+004
40	-6.64741666e+003	-6.6483980e+003
41	-1.07078558e+003	-1.0699068e+003
42	9.75881745e+002	9.7628697e+002
43	4.65862618e+002	4.6524643e+002
44	2.05599580e+002	2.0577343e+002
45	-1.33072574e+004	-1.3308121e+004
46	-4.46315490e+003	-4.4626558e+003
47	2.89792804e+004	2.8978946e+004
48	1.40603092e+004	1.4060198e+004
49	-2.25841617e+006	-2.2584170e+006
50	3.05459279e+006	3.0545938e+006
51	-1.06544644e+006	-1.0654473e+006
52	7.36273806e+005	7.3627455e+005
53	-1.45867623e+006	-1.4586752e+006
54	-2.41028655e+005	-2.4102947e+005
55	-8.83327271e+004	-8.8331883e+004
56	-3.08483326e+004	-3.0847822e+004
57	-1.42592640e+004	-1.4258353e+004
58	1.00050103e+004	1.0005582e+004
59	4.50901387e+003	4.5096899e+003
60	-1.26289924e+004	-1.2629970e+004
61	-2.17788367e+003	-2.1771588e+003
62	-8.88780002e+002	-8.8782196e+002
63	3.55786429e+002	3.5649279e+002
64	-8.64479364e+002	-8.6362237e+002
65	-2.94553098e+002	-2.9528026e+002
66	-1.19464412e+002	-1.2046090e+002
67	1.20133603e+001	1.1162413e+001
68	-1.14988925e+001	-1.1981227e+001
69	-5.11226611e+000	-5.5015713e+000
70	-2.24851827e+000	-1.6214982e+000
71	5.08868166e+000	4.7212066e+000

72	-6.23892970e+001	-6.3292957e+001
73	-1.80742766e+001	-1.8788502e+001
74	-7.02166790e+000	-7.7611588e+000
75	1.67248593e+001	1.7250868e+001
76	-9.86496882e+001	-9.8344172e+001
77	-4.82732765e+001	-4.7864229e+001
78	-2.32464019e+001	-2.2936069e+001
79	-1.14881019e+001	-1.1960956e+001
80	-5.12974823e+000	-5.5350960e+000
81	-2.23809411e+000	-1.6192284e+000
82	5.31064532e+000	4.7474428e+000
83	-2.20236066e+001	-2.1024133e+001
84	-1.66030868e+000	-1.5709158e+000
85	3.90719752e+002	3.9032249e+002
86	1.87183421e+002	1.8692764e+002
87	-5.43202467e+003	-5.4310468e+003
88	-9.41381437e+002	-9.4183828e+002
89	-4.42935582e+002	-4.4193600e+002
90	1.21891570e+001	1.1259462e+001
91	-5.63579594e+000	-6.4334569e+000
92	-1.62267598e+000	-1.5708196e+000
93	1.16588455e+003	1.1668227e+003
94	-6.19197514e+002	-6.1824330e+002
95	-1.43300466e+002	-1.4365087e+002
96	-6.91235980e+001	-7.0123561e+001
97	1.60536580e+000	1.6399284e+000
98	7.85156583e-001	7.7878999e-002
99	7.39531555e-001	7.4721112e-004
100	7.39085177e-001	7.3612660e-008
101	7.39085133e-001	7.7715612e-016

Approximate solution = 7.3908513322e-001

with $F(P) = 7.7715611724e-016$

Number of iterations = 101

Tolerance = 1.0000000000e-005

(d) Secant Method with $P_0 = 0$ and $P_1 = 1$.

I	P	F(P)
2	6.85073357e-01	-8.92992765e-02
3	7.36298998e-01	-4.66003904e-03
4	7.39119362e-01	5.72859911e-05
5	7.39085112e-01	-3.52926228e-08
6	7.39085133e-01	-2.66770113e-13

Approximate solution P = 0.73908513

with F(P) = -0.00000000

Number of iterations = 6 Tolerance = 1.00000000e-05

(e) Steffensen's Method with $g(x) = \cos x$ and $P_0 = 0.5$

I	P
1	7.31385186e-001
2	7.39076340e-001
3	7.39085133e-001

Approximate solution = 0.73908513

Number of iterations = 3

Tolerance = 1.00000000e-005

Conclusion: Yes, the number of the iterations decreases when the order of method increases.