

Answers to Homework 5, Math 4363(Fall 2009)

3.1

1a

(i)

$$\begin{aligned} P_1(x) &= \sum_{k=0}^1 f(x_k)L_{1,k}(x) = \cos(x_0)\frac{x-x_1}{x_0-x_1} + \cos(x_1)\frac{x-x_0}{x_1-x_0} \\ &= \cos(0)\frac{x-0.6}{0-0.6} + \cos(0.6)\frac{x-0}{0.6-0} = 1 + \frac{\cos(0.6)-1}{0.6}x \approx 1 - 0.2911073x. \end{aligned}$$

$$f(0.45) \approx P_1(0.45) \approx 0.86900171.$$

$$|f(0.45) - P_1(0.45)| \approx 0.03144539.$$

(ii)

$$\begin{aligned} P_2(x) &= \sum_{k=0}^2 f(x_k)L_{2,k}(x) \\ &= \cos(0)\frac{(x-0.6)(x-0.9)}{(0-0.6)(0-0.9)} + \cos(0.6)\frac{(x-0)(x-0.9)}{(0.6-0)(0.6-0.9)} + \cos(0.9)\frac{(x-0)(x-0.6)}{(0.9-0)(0.9-0.6)} \end{aligned}$$

$$\approx -0.43108687x^2 - 0.0324519x + 1.$$

$$f(0.45) \approx P_2(0.45) \approx 0.89810007.$$

$$|f(0.45) - P_2(0.45)| \approx 0.0023470.$$

3a

(i)

$$|f(0.45) - P_1(0.45)| = \left| \frac{f''(\xi)}{2} (0.45-0)(0.45-0.6) \right| \leq \frac{1}{2} \times 0.45 \times 0.15 = 0.03375,$$

since $|f''(x)| = |-\cos(x)| \leq 1$ for all $x \in (0,0.6)$.

(ii)

$$\begin{aligned} |f(0.45) - P_2(0.45)| &= \left| \frac{f'''(\xi)}{6} (0.45-0)(0.45-0.6)(0.45-0.9) \right| \\ &\leq \frac{\sin(0.9)}{6} \times 0.45 \times 0.15 \times 0.45 \approx 0.003965, \end{aligned}$$

since $|f'''(x)| = |\sin(x)| \leq \sin(0.9)$ for all $x \in (0,0.9)$.

17.

$$Q_{i,j}(x) = \frac{(x - x_{i-j})Q_{i,j-1} - (x - x_i)Q_{i-1,j-1}}{x_i - x_{i-j}}.$$

$$Q_{2,2}(0.5) = \frac{(0.5 - x_0)Q_{2,1} - (0.5 - x_2)Q_{1,1}}{x_2 - x_0} = \frac{27}{7},$$

$$\text{i.e. } \frac{(0.5 - 0)Q_{2,1} - (0.5 - 0.7) \times 3.5}{0.7 - 0} = \frac{27}{7},$$

$$\text{Then } Q_{2,1} = 4.$$

$$Q_{2,1}(0.5) = \frac{(0.5 - x_1)Q_{2,0} - (0.5 - x_2)Q_{1,0}}{x_2 - x_1} = 4,$$

$$\text{i.e. } \frac{(0.5 - 0.4)Q_{2,1} - (0.5 - 0.7) \times 2.8}{0.7 - 0.4} = 4,$$

$$\text{Then } Q_{2,0} = 6.4.$$

19.

(i)

$$\begin{aligned} P_3(x) &= \sum_{k=0}^3 f(x_k)L_{1,k}(x) \\ &= \cos(0) \frac{(x-0.25)(x-0.5)(x-1.0)}{(0-0.25)(0-0.5)(0-1.0)} + (\cos(0.25) + \sin(0.25)) \frac{(x-0)(x-0.5)(x-1.0)}{(0.25-0)(0.25-0.5)(0.25-1.0)} \\ &\quad + (\cos(0.5) + \sin(0.5)) \frac{(x-0)(x-0.25)(x-1.0)}{(0.5-0)(0.5-0.25)(0.5-1.0)} + (\cos(1.0) + \sin(1.0)) \frac{(x-0)(x-0.25)(x-0.5)}{(1.0-0)(1.0-0.25)(1.0-0.5)} \\ &\approx 1 + (-7 + 12.974041396961788 - 5.428032401978303 + 0.460591096892012)x + \\ &\quad (14 - 38.922124190885363 + 27.140162009891515 - 2.763546581352073)x^2 + \\ &\quad (-8 + 25.948082793923575 - 21.712129607913212 + 3.684728775136096)x^3 \\ &\approx 1 + 1.0066x - 0.5455x^2 - 0.07932x^3. \end{aligned}$$

(ii)

$$\begin{aligned} |f(x) - P_2(x)| &= \left| \frac{f^{(4)}(\xi)}{24} (x-0)(x-0.25)(x-0.5)(x-1.0) \right| \\ &\leq \frac{\sqrt{2}}{24} (1.0-0)^4 \approx 0.05893, \end{aligned}$$

$$\text{since } |f^{(4)}(x)| = \left| \sqrt{2} \sin\left(x + \frac{\pi}{4}\right) \right| \leq \sqrt{2} \quad \text{for all } x \in (0,1.0).$$