

Answers to Homework 6, Math 4363(Fall 2009)

3.2

3b

$$f[x_0] = -0.62049958,$$

$$f[x_0, x_1] = \frac{f[x_1] - f[x_0]}{x_1 - x_0} \approx 3.365129$$

$$f[x_1] = -0.28398668,$$

$$f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0} \approx -2.2962635$$

$$f[x_1, x_2] = \frac{f[x_2] - f[x_1]}{x_2 - x_1} \approx 2.9058763$$

$$f[x_2] = 0.00660095,$$

$$f[x_1, x_2, x_3] = \frac{f[x_2, x_3] - f[x_1, x_2]}{x_3 - x_1} \approx -2.438209$$

$$f[x_2, x_3] = \frac{f[x_3] - f[x_2]}{x_3 - x_2} \approx 2.4182345$$

$$f[x_3] = 0.24842440$$

$$f[x_0, x_1, x_2, x_3] = \frac{f[x_1, x_2, x_3] - f[x_0, x_1, x_2]}{x_3 - x_0} \approx -0.473151666,$$

$$P_1(x) = -0.62049958 + 3.365129(x - 0.1),$$

$$P_2(x) = P_1(x) - 2.2962635(x - 0.1)(x - 0.2), \quad P_2(0.25) \approx -0.13295217,$$

$$P_3(x) = P_2(x) - 0.473151666(x - 0.1)(x - 0.2)(x - 0.3), \quad P_3(0.25) \approx -0.132774738.$$

Note that $P_1(x)$ above is based on $(x_0, f[x_0])$ and $(x_1, f[x_1])$, and $0.25 \in (x_1, x_2) = (0.2, 0.3)$. So it is not proper

for approximating $f(0.25)$. We should use $P_1(x) = f[x_1] + f[x_1, x_2](x - x_1)$ instead, which is

$$P_1(x) = -0.28398668 + 2.9058763(x - 0.2),$$

and

$$P_1(0.25) = -0.138693.$$

7.

(a)

$$f[x_0] = 5.30000$$

$$f[x_0, x_1] = \frac{f[x_1] - f[x_0]}{x_1 - x_0} = -33.00000$$

$$f[x_1] = 2.00000,$$

$$f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0} \approx 129.83333$$

$$f[x_1, x_2] = \frac{f[x_2] - f[x_1]}{x_2 - x_1} \approx 5.95000$$

$$f[x_2] = 3.19000,$$

$$f[x_1, x_2, x_3] = \frac{f[x_2, x_3] - f[x_1, x_2]}{x_3 - x_1} \approx -92.83333$$

$$f[x_2, x_3] = \frac{f[x_3] - f[x_2]}{x_3 - x_2} \approx -21.90000$$

$$f[x_3] = 1.00000$$

$$f[x_0, x_1, x_2, x_3] = \frac{f[x_1, x_2, x_3] - f[x_0, x_1, x_2]}{x_3 - x_0} \approx -556.66667,$$

$$P_3(x) = 5.3 - 33(x + 0.1)(x - 0.1) + 129.83333(x + 0.1)x - 556.66667(x + 0.1)x(x - 0.2)$$

(b)

$$f[x_0] = 5.30000$$

$$f[x_0, x_1] = \frac{f[x_1] - f[x_0]}{x_1 - x_0} = -33.00000$$

$$f[x_1] = 2.00000,$$

$$f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0} \approx 129.83333$$

$$f[x_1, x_2] = \frac{f[x_2] - f[x_1]}{x_2 - x_1} \approx 5.95000$$

$$f[x_2] = 3.19000,$$

$$f[x_1, x_2, x_3] = \frac{f[x_2, x_3] - f[x_1, x_2]}{x_3 - x_1} \approx -92.83333$$

$$f[x_2, x_3] = \frac{f[x_3] - f[x_2]}{x_3 - x_2} \approx -21.90000$$

$$f[x_3] = 1.00000$$

$$f[x_2, x_3, x_4] = \frac{f[x_3, x_4] - f[x_2, x_3]}{x_4 - x_2} \approx 142.34667$$

$$f[x_3, x_4] = \frac{f[x_4] - f[x_3]}{x_4 - x_3} \approx -0.54800$$

$$f[x_4] = 0.97260$$

$$f[x_0, x_1, x_2, x_3] = \frac{f[x_1, x_2, x_3] - f[x_0, x_1, x_2]}{x_3 - x_0} \approx -556.66667,$$

$$f[x_1, x_2, x_3, x_4] = \frac{f[x_2, x_3, x_4] - f[x_1, x_2, x_3]}{x_4 - x_1} \approx 671.9428569,$$

$$f[x_0, x_1, x_2, x_3, x_4] = \frac{f[x_1, x_2, x_3, x_4] - f[x_0, x_1, x_2, x_3]}{x_4 - x_0} \approx 2730.243387.$$

$$P_4(x) = P_3(x) + 2730.243387(x + 0.1)x(x - 0.2)(x - 0.3)$$

$$f[x_1, x_2, x_3] = \frac{f[x_2, x_3] - f[x_1, x_2]}{x_3 - x_1} = \frac{\frac{f[x_3] - f[x_2]}{x_3 - x_2} - \frac{f[x_2] - f[x_1]}{x_2 - x_1}}{x_3 - x_1}$$

$$= \frac{f[x_3](x_2 - x_1) + f[x_2](x_1 - x_3) + f[x_1](x_3 - x_2)}{(x_2 - x_1)(x_3 - x_1)(x_3 - x_2)}$$

$$f[x_2, x_1, x_3] = \frac{f[x_1, x_3] - f[x_2, x_1]}{x_3 - x_2} = \frac{\frac{f[x_3] - f[x_1]}{x_3 - x_1} - \frac{f[x_1] - f[x_2]}{x_1 - x_2}}{x_3 - x_2}$$

$$= \frac{f[x_3](x_2 - x_1) + f[x_2](x_1 - x_3) + f[x_1](x_3 - x_2)}{(x_2 - x_1)(x_3 - x_1)(x_3 - x_2)}$$

$$f[x_1, x_3, x_2] = \frac{f[x_3, x_2] - f[x_1, x_3]}{x_2 - x_1} = \frac{\frac{f[x_2] - f[x_3]}{x_2 - x_3} - \frac{f[x_3] - f[x_1]}{x_3 - x_1}}{x_2 - x_1}$$

$$= \frac{f[x_3](x_2 - x_1) + f[x_2](x_1 - x_3) + f[x_1](x_3 - x_2)}{(x_2 - x_1)(x_3 - x_1)(x_3 - x_2)}$$

So

$$f[x_1, x_2, x_3] = f[x_2, x_1, x_3] = f[x_1, x_3, x_2].$$