Answers to Homework 7, Math 4363(Fall 2009)

ex3.4

3d.

FREE CUBIC SPLINE INTERPOLATION

The numbers X(0), ..., X(N) are:

0.10000000 0.20000000 0.30000000 0.40000000

The coefficients of the spline on the subintervals are:

for I = 0, ..., N-1

A(I)	B(I)	C(I)	D(I)
-0.62049958	3.45508693	0.00000000	-8.99579333
-0.28398668	3.18521313	-2.69873800	-0.94630333
0.00660095	2.61707643	-2.98262900	9.94209667

So

$$S(x) = \begin{cases} -0.62049958 + 3.45508693(x-0.1) - 8.99579333(x-0.1)^3, & if\ 0.1 \le x < 0.2 \\ -0.28398668 + 3.18521313(x-0.2) - 2.698738(x-0.2)^2 - 0.94630333(x-0.2)^3, & if\ 0.2 \le x < 0.3 \\ 0.00660095 + 2.61707643(x-0.3) - 2.98262900(x-0.3)^2 + 9.942096677(x-0.3)^3, & if\ 0.3 \le x < 0.4 \end{cases}$$

5d.

Approximation

$$f(0.25) \approx -0.28398668 + 3.18521313(0.25-0.2) - 2.698738(0.25-0.2)^2 - 0.94630333(0.25-0.2)^3$$

$$\approx -0.1316$$

actual value

$$f(0.25) = 0.25\cos 0.25 - 2(0.25)^2 + 3(0.25) - 1$$

$$\approx -0.1328$$

Error

$$\left|-0.1316 - (-0.1328)\right| = 0.0012.$$

Approximation

$$f'(0.25) \approx 3.18521313 - 2 \times 2.698738(0.25-0.2) - 3 \times 0.94630333(0.25-0.2)^2$$

$$\approx 2.9082$$

actual value

$$f'(x) = \cos x - x\sin x - 4x + 3$$

$$f'(0.25) = \cos 0.25 - 0.25\sin(0.25) - 4(0.25) + 3$$

$$\approx 2.9071$$

Error

$$\left|2.9071 - 2.9082\right| = 0.0011.$$

11.

$S_0'(x) = 2 - 3x^2,\qquad S_0''(x) = -6x,$

$S_0'(1) = -1,\qquad S_0''(1) = -6,\qquad S_0''(0) = 0,$

$S_1'(x) = b + 2c(x-1) + 3d(x-1)^2,\qquad S_1''(x) = 2c + 6d(x-1)$

$S_1'(1) = b,\qquad S_1''(1) = 2c,\qquad S_1''(2) = 2c + 6d$

$S_0'(1) = S_1'(1),\quad S_1''(1) = S_0''(1),\quad S_0''(0) = S_1''(2)$

So

$b = -1,\ c = -3, d = 1.$

14.

$S_0'(x) = B + 4x - 6x^2,\ S_0''(x) = 4 - 12x,$

$S_0'(1) = B - 2,\quad S_0(1) = 1 + B$

$S_1'(x) = b - 8(x-1) + 21(x-1)^2,\qquad S_1''(x) = -8 + 42(x-1)$

$S_1'(1) = b,\qquad S_1(1) = 1$

$S_0(1) = S_1(1),\quad S_0'(1) = S_1'(1),$

Then

$B = 0,\quad b = -2$

So

$f'(0) = S_0'(0) = 0,\qquad f'(2) = S_1'(2) = -2 - 8(2-1) + 21(2-1)^2 = 11.$

ex4.1

Forward-difference formula

$$f'(x_0) \approx \frac{f(x_0+h)-f(x_0)}{h}, \quad h>0$$

$$f'(0.5) \approx \frac{f(0.6)-f(0.5)}{0.1} = \frac{0.5646-0.4794}{0.1} = 0.852$$

$$f'(0.6) \approx \frac{f(0.7)-f(0.6)}{0.1} = \frac{0.6442-0.5646}{0.1} = 0.796$$

Backward-difference formula

$$f'(x_0) \approx \frac{f(x_0+h)-f(x_0)}{h}, \quad h<0$$

$$f'(0.7) \approx \frac{f(0.6)-f(0.7)}{-0.1} = \frac{0.5646-0.6442}{-0.1} = 0.796.$$

Or

$$f'(0.5) = -0.852, \qquad f'(0.6) = 0.852, \qquad f'(0.7) = 0.796$$

3a.

Error bound :

$$\left|\frac{-hf''(\xi)}{2}\right| \qquad where\ f''(x) = -\sin x, \qquad h = 0.1$$

So for $x \in [0.5,0.6]$, $\left|\frac{-hf''(\xi)}{2}\right| \le \frac{0.1}{2}\times\sin(0.6) \approx 0.028232$

So for $x \in [0.6,0.7]$, $\left|\frac{-hf''(\xi)}{2}\right| \le \frac{0.1}{2}\times\sin(0.7) \approx 0.032211$

x	*actual value*	*approximation*	*actual error*	*error bound*
0.5	0.87758	0.852	0.02558	0.028232
0.6	0.82533	0.852	0.02666	0.028232
0.7	0.76484	0.796	0.03116	0.032211

5a.

For $f'(1.1)$ *and* $f'(1.4)$, *we* use formula (4.4)

For $f'(1.2)$ *and* $f'(1.3)$, *we* use formula (4.5).

$f'(1.1) \approx 17.769705,$
$f'(1.2) \approx 22.193635,$
$f'(1.3) \approx 27.107350,$
$f'(1.4) \approx 32.510850.$

7a.
Error bound :

$f'''(x) = 8e^{2x}, \quad h = 0.1$

when $x = 1.1, \xi_0$ *lies between* 1.1 *and* 1.3, *so*

$$\left|\frac{-h^2 f'''(\xi_0)}{3}\right| \le \frac{0.01}{3} \times 8e^{2(1.3)} \approx 0.359033,$$

when $x = 1.2,$ ξ_1 *lies between* 1.1 *and* 1.3, *so*

$$\left|\frac{-h^2 f'''(\xi_1)}{6}\right| \le \frac{0.01}{6} \times 8e^{2(1.3)} \approx 0.179517,$$

when $x = 1.3,$ ξ_1 *lies between* 1.2 *and* 1.4, *so*

$$\left|\frac{-h^2 f'''(\xi_1)}{6}\right| \le \frac{0.01}{6} \times 8e^{2(1.4)} \approx 0.219262$$

when $x = 1.4,$ ξ_0 *lies between* 1.2 *and* 1.4, *so*

$$\left|\frac{-h^2 f'''(\xi_0)}{3}\right| \le \frac{0.01}{3} \times 8e^{2(1.4)} \approx 0.438524.$$

x	*actual value*	*approximation*	*actual error*	*error bound*
1.1	18.050027	17.769705	0.280322	0.359033
1.2	22.046353	21.193635	0.147282	0.179517
1.3	26.927476	27.107350	0.179874	0.219262
1.4	32.889294	32.510850	0.378444	0.438524