MATH 4364: Numerical Analysis
Fall 2009

Computer Project # 1

**Project due on November 11.**: Please turn in programs, figures, and results.

1. Consider the function $g_1(x) = \sin(\pi x)$ on the interval $[-1, 1]$. Use Algorithm 3.2 to obtain the coefficients of the Newton's divided-difference polynomial $P_n(x)$ given in (3.10) for the function $g_1(x)$ on the following sets of equally spaced nodes

$$x_i = -1 + \frac{2i}{n}, \text{ for } i = 0, 1, 2, \ldots, n,$$

for $n = 4, 6, 8$ and $10$, respectively. Plot graphs of $g_1(x)$ and its interpolating polynomials associated with above given sets of nodes. Does Lagrange interpolating polynomial approximate $g_1(x)$ better when $n$ is larger? Why?

2. Consider the function $g_2(x) = 1/(1 + x^2)$ on the interval $[-5, 5]$.

a. Use Algorithm 3.2 to obtain the coefficients of the Newton's divided-difference polynomial $P_n(x)$ given in (3.10) for the function $g_2(x)$ on the following two sets of nodes,

(i) equally spaced nodes:

$$x_i = -5 + \frac{10i}{n}, \quad i = 0, 1, 2, \ldots, n,$$

(ii) Chebyshev nodes:

$$x_i = 5\cos\left(\frac{2i+1}{2n+2}\pi\right), \quad i = 0, 1, 2, \ldots, n,$$

for $n = 8$, respectively. Plot graphs of $g_2(x)$ and its interpolating polynomials associated with above given sets of nodes. Does Lagrange interpolating polynomial based on the Chebyshev nodes approximate $g_2(x)$ better?

b. Use Algorithm 3.5 to find a clamped cubic spline interpolant $S(x)$ for $g_2(x)$ on the nodes

$$x_i = -5 + \frac{10i}{n}, \quad i = 0, 1, \ldots, n$$

for $n = 8$ and $g_2'(-5) = 5/338$ and $g_2'(5) = -5/338$. Does the cubic spline interpolant $S(x)$ give the smallest error among the above three approximations?