Math 3363 Spring 2020

Homework 1

Due January 21 at 2:30 p.m.

Please use a pencil and do the problems in the order in which they are listed. Write on only one side of each page and staple your pages.

1. Do Problem 1.4.1(b) on page 18. The problem to solve is

$$\begin{aligned} u''(x) &= 0 \text{ for } 0 \le x \le L, \\ u(0) &= T, \text{ and} \\ u(L) &= 0. \end{aligned}$$

2. Do Problem 1.4.1(e) on page 18. The problem to solve is

$$\begin{aligned} K_0 u''(x) + Q &= 0 \text{ for } 0 \le x \le L, \\ u(0) &= T_1 \text{ and} \\ u(L) &= T_2 \end{aligned}$$

   when

$$\frac{Q}{K_0} = 1.$$

3. Do Problem 1.4.1(g) on page 18. The problem to solve is

$$\begin{aligned} u''(x) &= 0 \text{ for } 0 \le x \le L, \\ u(0) &= T, \text{ and} \\ u'(L) + u(L) &= 0. \end{aligned}$$

4. Do Problem 1.4.2(a) on page 18 of the text. You are being told that $Q(x) = K_0 x$ is the heat per unit time per unit length being generated at coordinate x for $0 \le x \le L$.

5. Do Problem 1.4.2(b) on page 18 of the text. The differential equation to solve is

$$K_0 u''(x) + Q(x) = 0.$$

   K_0 is a constant and heat flowing to the right at coordinate x is $-K_0 u'(x)$. Note that heat flowing out at the left end of the rod is $-(-K_0 u'(0)) = K_0 u'(0)$.

6. Do Problem 1.4.2(c) on page 18 of the text.

7. Let

$$Q(x) = 10x - x^2$$

and let

$$f(x) = |x - 5| - 5 \text{ for } 0 \leq x \leq 10.$$

Find the value of β so that the following problem has an equilibrium solution.

$$\frac{\partial w}{\partial t}(x,t) = \frac{\partial^2 w}{\partial x^2}(x,t) + Q(x) \text{ for } 0 \leq x \leq 10 \text{ and } t \geq 0,$$

$$\frac{\partial w}{\partial x}(0,t) = \beta \text{ for } t \geq 0 \ ,$$

$$\frac{\partial w}{\partial x}(10,t) = 1 \text{ for } t \geq 0, \text{ and}$$

$$w(x,0) = f(x) \text{ for } 0 \leq x \leq 10.$$

The related equilibrium problem is stated below in Problem 8.

8. Find the equilibrium solution u for Problem 7. The problem for u is

$$0 = u''(x) + Q(x) \text{ for } 0 \leq x \leq 10,$$

$$u'(0) = \beta, \text{ and}$$

$$u'(10) = 1.$$

Use the value for β that you found in problem 7. There should be no undetermined constant in your answer. You may assume that

$$\lim_{t\to\infty} w(x,t) = u(x)$$

for $0 \leq x \leq 10$, and you may interchange limits and integrals when you need to do this.