Math 3363 Spring 2020

Homework 2

Due February 4 at 2:30 p.m.

Please use a pencil and do the problems in the order in which they are listed. Write on only one side of each page and staple your pages.

1. Use the Rayleigh Quotient to show that all eigenvalues for

$$\begin{aligned} -\varphi''(x) &= \lambda\varphi(x) \text{ for } 0 \le x \le L, & (1) \\ \varphi'(0) &= 0, \text{ and} & (2) \\ \varphi(L) &= 0. & (3) \end{aligned}$$

   are non-negative. Note that L is a positive number.

2. Show that zero is not an eigenvalue for (1), (2), and (3).

3. Find the matrix $D(\lambda)$ for (1), (2), and (3) when $\lambda > 0$.Note that (2) and (3) are equivalent to

$$\begin{aligned} \beta_1\varphi(0) + \beta_2\varphi'(0) &= 0 \text{ and} \\ \beta_3\varphi(L) + \beta_4\varphi'(L) &= 0 \end{aligned}$$

   where $\beta_1 = 0$, $\beta_2 = 1$, $\beta_3 = 1$, and $\beta_4 = 0$.

4. Find the function $\Delta(\lambda)$ for (1), (2), and (3) when $\lambda > 0$ then find the eigenvalues.

5. For each eigenvalue for (1), (2), and (3), find the corresponding eigenfunctions.

6. A proper listing of eigenvalues and eigenfunctions for

$$\begin{aligned} -\varphi''(x) &= \lambda\varphi(x) \text{ for } 0 \le x \le L, \\ \varphi'(0) &= 0, \text{ and} \\ \varphi'(L) &= 0 \end{aligned}$$

   is $\{\lambda_k\}_{k=0}^{\infty}$ and $\{\varphi_k\}_{k=0}^{\infty}$ where $\lambda_0 = 0$, $\varphi_0(x) = 1$ and $\lambda_k = \left(\dfrac{k\pi}{L}\right)^2$ and $\varphi_k(x) = \cos\dfrac{k\pi x}{L}$ for $k = 1, 2, \ldots$. Let

$$f(x) = 2x^3 - 3Lx^2 \text{ for } 0 \le x \le L.$$

   Find the series for f determined by the orthogonal sequence $\{\varphi_k\}_{k=0}^{\infty}$. Evaluate the integrals. The integration by parts formula is

$$\int_a^b u(x)v'(x)dx = [u(x)v(x)]_{x=a}^{x=b} - \int_a^b u'(x)v(x)dx$$

   whenever each of u and v has a continuous derivative on $[a, b]$.