Mathematics 3363
Review for Examination II

Spring 2020

1. Study your class notes.

2. Study the notes on Dr. Walker's web site.

3. Review suggested homework problems and Homeworks 3 and 4.

4. Suppose that each of L and H is a positive number. Derive the solution to

$$\begin{aligned}
\frac{\partial^2 u}{\partial x^2}(x,y) + \frac{\partial^2 u}{\partial y^2}(x,y) &= 0 \text{ for } 0 \le x \le L \text{ and } 0 \le y \le H, \\
u(0,y) &= u(L,y) = 0 \text{ for } 0 \le y \le H, \\
u(x,0) &= f(x), \text{ and } u(x,H) = 0 \text{ for } 0 \le x \le L.
\end{aligned}$$

5. Suppose that each of L and H is a positive number. Derive the solution to

$$\begin{aligned}
\frac{\partial^2 u}{\partial x^2}(x,y) + \frac{\partial^2 u}{\partial y^2}(x,y) &= 0 \text{ for } 0 \le x \le L \text{ and } 0 \le y \le H, \\
u(0,y) &= u(L,y) = 0 \text{ for } 0 \le y \le H, \\
u(x,H) &= f(x), \text{ and } u(x,0) = 0 \text{ for } 0 \le x \le L.
\end{aligned}$$

6. Suppose that each of L and H is a positive number. Derive the solution to

$$\begin{aligned}
\frac{\partial^2 u}{\partial x^2}(x,y) + \frac{\partial^2 u}{\partial y^2}(x,y) &= 0 \text{ for } 0 \le x \le L \text{ and } 0 \le y \le H, \\
u(0,y) &= f(y) \text{ and } u(L,y) = 0 \text{ for } 0 \le y \le H \\
u(x,H) &= 0 \text{ and } u(x,0) = 0 \text{ for } 0 \le x \le L.
\end{aligned}$$

7. Suppose that each of L and H is a positive number. Derive the solution to

$$\begin{aligned}
\frac{\partial^2 u}{\partial x^2}(x,y) + \frac{\partial^2 u}{\partial y^2}(x,y) &= 0 \text{ for } 0 \le x \le L \text{ and } 0 \le y \le H, \\
u(0,y) &= 0 \text{ and } u(L,y) = f(y) \text{ for } 0 \le y \le H, \\
u(x,H) &= 0 \text{ and } u(x,0) = 0 \text{ for } 0 \le x \le L.
\end{aligned}$$

8. Find the function v of the form

$$v(x,y) = ax + by + cxy + d$$

such that

$$\begin{aligned}
v(0,0) &= -1, \\
v(2,0) &= 3, \\
v(2,4) &= 4, \text{ and} \\
v(0,4) &= -2.
\end{aligned}$$

9. Derive the solution to

$$\begin{aligned}
\frac{\partial^2 u}{\partial x^2}(x,y) + \frac{\partial^2 u}{\partial y^2}(x,y) &= 0 \text{ for } 0 \le x \le 2 \text{ and } 0 \le y \le 1, \\
u(0,y) &= y^2 + y + 4 \text{ and } u(2,y) = 8y + 6 \text{ for } 0 \le y \le 1, \\
u(x,1) &= 4x + 6 \text{ and } u(x,0) = x^2 - x + 4 \text{ for } 0 \le x \le 2.
\end{aligned}$$

In order to improve the convergence of the series solution, do this by first finding a function v of the form

$$v(x,y) = ax + by + cxy + d$$

that agrees with the given boundary conditions at the four corners of the rectangle. Then let

$$w(x,y) = u(x,y) - v(x,y)$$

for all (x,y)in the rectangle. Calculate the boundary conditions for w (w will be zero at the four corners) and noting that w is also a solution to Laplace's equation find the function w. Find u by noting that $u = w + v$.

10. Do problems 2.5.1 and 2.5.2 in the text.

11. State the maximum principle for solutions to Laplace's equation.

12. Suppose that $L \geq 0$ and $H \geq 0$. Show that there is at most one function u that is continuous on the rectangle consisting of all (x, y) where $0 \leq x \leq L$ and $0 \leq y \leq H$ and satisfies Laplace's equation on the interior of this rectangle.

13. Do the following problems from the text. When Haberman asks you to sketch a series, he should be asking you to sketch the limit of that series, and that is what you are to do.

    (a) 3.2.1 and 3.2.2 page 92.

    (b) 3.3.1 through 3.3.5.

14. Let
$$f(x) = \begin{cases} 1 - x & \text{when} \quad -1 < x < 0 \\ 0 & \text{when} \quad 0 < x < 1 \end{cases}$$
Find the Fourier series for f.

15. Let $\{S_n\}$ be the Fourier series for the function f in the previous problem and let
$$g(x) = \lim_{n \to \infty} S_n(x) \text{ for } -5 \leq x \leq 5.$$
Sketch the graph of g. Be sure to indicate the value of g at the numbers where g is discontinuous.

16. Suppose that L is a positive number and f is piecewise continuous.

    (a) Define the Fourier series for f when f is defined on $[-L, L]$.

    (b) Define the cosine series for f when f is defined on $[0, L]$.

    (c) Define the sine series for f when f is defined on $[0, L]$.

17. Suppose that L is a positive number and that j and k are nonnegative integers ($j \neq k$). Evaluate
$$\int_{-L}^{L} \cos \frac{k\pi x}{L} \cos \frac{j\pi x}{L} dx.$$

18. Let

$$f(x) = \begin{cases} -1 & \text{when} \quad -1 < x < 0 \\ 1 & \text{when} \quad 0 < x < 1 \end{cases}, .$$

and let $\{S_n\}_{n=1}^{\infty}$ be the Fourier series for f. Sketch the graph of f and the graph of a typical S_n over $[-\frac{1}{2}, \frac{1}{2}]$ on the same set of axes. Describe the Gibbs phenomenon.

19. Let f and $\{S_n\}_{n=1}^{\infty}$ be as in the previous problem. Explain why $\{S_n\}_{n=1}^{\infty}$ does not converge uniformly.

20. Show that the only function that is both even and odd is the zero function.

21. Let $f(x) = 1 - x^2$ for $0 < x < 1$.

    (a) Sketch the function to which the cosine series of f converges on $[-4, 4]$.

    (b) Sketch the function to which the sine series of f converges on $[-4, 4]$.

22. Suppose that each of c and L is a positive number. Derive the solution to

$$\begin{aligned} \frac{\partial^2 u}{\partial t^2}(x,t) &= c^2 \frac{\partial^2 u}{\partial x^2}(x,t) \text{ for } 0 \le x \le L \text{ and all } t \text{ in } \mathbb{R}, \\ u(0,t) &= 0 \text{ for all } t \text{ in } \mathbb{R}, \\ u(L,t) &= 0 \text{ for all } t \text{ in } \mathbb{R}, \\ u(x,0) &= f(x) \text{ for } 0 \le x \le L, \text{ and} \\ \frac{\partial u}{\partial t}(x,0) &= g(x) \text{ for } 0 \le x \le L. \end{aligned}$$

23. Suppose that each of c and L is a positive number. Derive the solution

to

$$\begin{aligned}
\frac{\partial^2 u}{\partial t^2}(x,t) &= c^2 \frac{\partial^2 u}{\partial x^2}(x,t) \text{ for } 0 \le x \le L \text{ and all } t \text{ in } \mathbb{R}, \\
u(0,t) &= 0 \text{ for all } t \text{ in } \mathbb{R}, \\
\frac{\partial u}{\partial x}(L,t) &= 0 \text{ for all } t \text{ in } \mathbb{R}, \\
u(x,0) &= f(x) \text{ for } 0 \le x \le L, \text{ and} \\
\frac{\partial u}{\partial t}(x,0) &= g(x) \text{ for } 0 \le x \le L.
\end{aligned}$$

24. Suppose that the speed of sound in air is 343 m/s and an organ pipe that is closed at one end and open at the other sounds with a fundamental frequency of 110 Hz. Find the length of the pipe.