MATH 3363 MIDTERM EXAM 3. Sanders Fall 2005

This exam has 5 problems and all 5 problems will be graded. You have the full hour and a half to complete it. Use my supplied paper only and return your solution sheets with the problems in order. Put your name, **last name first**, and **social security number** on each solution sheet you turn in. Good luck.

1a. Derive the 2π periodic Fourier series for the function $f(\theta) = \theta$. (Answer: On the interval $-\pi < \theta < \pi$, $\theta = \sum_{n=1}^{\infty} -2\frac{(-1)^n}{n}\sin(n\theta)$.)

1b. Use Parseval and the answer in part (a) to compute the value of the sum $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

2a. Solve Laplace's equation $\nabla^2 u = 0$ on the disk $r < 1$ with boundary condition $u(1,\theta) = \theta$ for $-\pi < \theta < \pi$.

2b. Solve Laplace's equation $\nabla^2 u = 0$ on the annular region $1 < r < 2$ with boundary conditions $u(1,\theta) = 2$, $u(2,\theta) = 1$.

3a. Use the solution $u(z)$ to the differential equation $\frac{d^2u}{dz^2} + z^2\frac{du}{dz} = 0$ to determine a solution to $\frac{d^2y}{dx^2} + 5x^2\frac{dy}{dx} = 0$.

3b. Suppose $u(z)$ solves $\frac{d^2u}{dz^2} = \lambda u$ and boundary conditions $u(0) = 0$, $u(1) = 0$. What boundary value problem (differential equation and boundary conditions) does $y(x) \equiv u(\frac{1}{2}x)$ solve?

4. Use the Fourier transform technique to solve the following first order equations on the interval $-\infty < x < \infty$. (Your final answer must not involve the Fourier transform of f.)

(a) $\frac{\partial u}{\partial t} + 4\frac{\partial u}{\partial x} = 0$, $u(x,0) = f(x)$.

(b) $\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} + 2u = 0$, $u(x,0) = f(x)$.

5a. (15 points) Derive D'Alembert's solution to the wave equation on the bounded interval $0 < x < 1$

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0 \quad \begin{array}{ll} \text{with:} & u(0,t) = 0,\ u(1,t) = 0 \text{ for } t > 0. \\ \text{and:} & u(x,0) = f(x),\ u_t(x,0) = 0 \text{ for } 0 < x < 1. \end{array}$$

(Answer: $u(x,t) = \frac{1}{2}(f^o(x+t) + f^o(x-t))$ where f^o denotes the odd extension of f.)

5b. (5 points) Graphically depict the solution at $t = 1/4$ for the initial data, $f(x)$, I give on the board.