MATH 3363 MIDTERM EXAM II. Sanders Fall 2006

This exam has 5 problems, and all 5 problems will be graded. Use my supplied paper only. Return your solution sheets with the problems in order. Put your name, **last name first**, and **student id number** on each solution sheet you turn in. Each problem is worth 20 points with parts equally weighted unless otherwise indicated.

1. Solve the heat equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ on $0 \le x \le 1,\ 0 \le t$ with the given boundary/initial conditions.

(a) $u(0,t) = 0,\ u(1,t) = 0$, $u(x,0) = \sin(\pi x) + 2\sin(2\pi x)$ (b) $u_x(0,t) = 0,\ u_x(1,t) = 0$, $u(x,0) = 1 + \cos(\pi x)$

2. Consider the heat equation with inhomogeneous boundary conditions.

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2},$$

with boundary conditions: $u(0,t) = 1,\ u(1,t) = 0$,
and initial condition: $u(x,0) = \sin(\pi x) - x + 1$.

(a) Determine the steady-state solution.

(b) Solve for $u(x,t)$.

3. Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ on the unit interval $0 < x < 1$ subject to following boundary and initial conditions.

(a) $u_x(0,t) = 0 \quad u_x(1,t) = 0$, $u(x,0) = \cos(\pi x) + 2\cos(2\pi x)$, $u_t(x,0) = 0$ (b) $u_x(0,t) = 0 \quad u_x(1,t) = 0$, $u(x,0) = 0$, $u_t(x,0) = 1$

4. Solve Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ on the unit square $0 \le x \le 1,\ 0 \le y \le 1$ subject to the given boundary conditions.

(a) $u_x(0,y) = 0$, $u_x(1,y) = 0$, $u(x,0) = 1$, $u(x,1) = 0$ (b) $u_x(0,y) = 0$, $u_x(1,y) = \sin(\pi y)$, $u(x,0) = 0$, $u(x,1) = 0$

5. Find the solution of Laplace's equation $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$ on the following domains satisfying the given boundary conditions.

(a) The unit disk $r \le 1$ with $u(1,\theta) = 1 + \cos(\theta)$. (b) The annulus $1 \le r \le 2$ with $u(1,\theta) = 1 + \sin(\theta)$, and $u(2,\theta) = 1$.