Q1.1

#2 on fall 03 Exam 2

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - 2$$

$u(0,t) = 1 \qquad u(1,t) = 0$

$u(x,0) = x^2 - x + 1$

(*) $u(x,t) = \underline{S(x) + v(x,t)}$

$$0 = \frac{d^2 S}{dx^2} - 2$$

$S(0) = 1 \qquad S(1) = 0$

2 point BVP

$$\boxed{S(x) = a + bx + x^2}$$

$1 = S(0) = a \qquad a = 1$

$0 = S(1) = a + b + 1 \qquad b = -2$

$$\underline{S(x) = 1 - 2x + x^2}$$

Q1.2

$$\boxed{\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial x^2}}$$

$$\boxed{V(0,t) = 0 \quad V(1,t) = 0}$$

$$V(X,0) = u(x,0) - S(x) \quad (\text{Use } *)$$

New I.C. $= \cancel{(x^2 - x + 1)} - \cancel{(1 - 2x + x^2)}$

$$\underline{V(x,0) = x}$$

$$\boxed{V(x,t) = \sum_{n=1}^{\infty} e^{-(n\pi)^2 t} \underline{a_n} \sin(n\pi x)}$$

$$V(x,0) = \sum_{n=1}^{\infty} a_n \sin(n\pi x) = x$$

$$x = \sum_{n=1}^{\infty} \gamma_n \sin(n\pi x)$$

$$\gamma_n = 2\int_0^1 \overset{u}{x} \, \overset{dv}{\sin(n\pi x)\,dx}$$

$$= 2\left[-x \frac{\cos(n\pi x)}{n\pi}\Big|_0^1 + \int_0^1 \frac{\cos(n\pi x)}{n\pi} \right] = 2\frac{(-\cos(n\pi))}{n\pi}$$

Q 1.3

$$v(x,t) = -\sum_{n=1}^{\infty} e^{-(n\pi)^2 t}\, \frac{2(-1)^n}{n\pi} \sin(n\pi x)$$

$$\boxed{u(x,t) = \underbrace{(1-2x+x^2)}_{S(x)} \underbrace{- \sum_{n=1}^{\infty} e^{-(n\pi)^2 t}\, \frac{2(-1)^{n+1}}{n\pi} \sin(n\pi x)}_{v(x,t)}}$$

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} + 2u = 0$$

Q2,1

#4b on Sp08 Exam 3

$$-\infty < X < \infty$$

$$u(x,0) = f(x)$$

$$u(x,t) = \int_{-\infty}^{\infty} \hat{u}(\omega,t) e^{-i\omega x} d\omega$$

$$\frac{d\hat{u}}{dt} - i\omega \hat{u} + 2\hat{u} = 0$$

$$\frac{d\hat{u}}{dt} + (2 - i\omega)\hat{u} = 0$$

Solve This to get

$$\hat{u}(\omega,t) = e^{-(2-i\omega)t} \underbrace{\boxed{\hat{u}(\omega,0)}}_{\text{Constant @ } t=0}$$

$$\hat{u}(\omega,0) = \hat{f}(\omega)$$

$$u(x,t) = \int_{-\infty}^{\infty} e^{-(2-i\omega)t} \hat{f}(\omega) e^{-i\omega x} d\omega$$

Q2.2

$$u(x,t) = e^{-2t}\int_{-\infty}^{\infty} \hat{f}(\omega)\, e^{-i\omega(x-t)}\, d\omega$$

$$f(\theta) = \int_{-\infty}^{\infty} \hat{f}(\omega)\, e^{-i\omega\theta}\, d\omega$$

Recover f from the inverse F.T of its F.T.

$$\boxed{u(x,t) = e^{-2t} f(x-t)}$$

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} + 2u = 0$$

$$-\infty < x < \infty$$

$$u(x,0) = \frac{1}{1+x^2}$$

$$u(x,t) = e^{-2t}\frac{1}{1+(x-t)^2}$$

$$\boxed{\frac{d^2u}{dz^2} + z\frac{du}{dz} = 0}$$ $u(z)$

Sp 08
Exam 3
#2a

$$\frac{d^2v}{dx^2} + 10x\frac{dv}{dx} = 0 \quad \checkmark$$

$z = \alpha x$ $\qquad$ $v(x) = u(z)$

$\frac{z}{\alpha}$

$$\alpha^2\frac{d^2u}{dz^2} + 10\frac{z}{\cancel{\alpha}}\cancel{\alpha}\frac{du}{dz} = 0$$

$$= \alpha^2\frac{d^2u}{dz^2} + 10z\frac{du}{dz} = 0$$

$$\boxed{\alpha = \sqrt{10}}$$

$$\frac{d^2u}{dz^2} + \cancel{\frac{10}{\alpha^2}}z\frac{du}{dz} = 0$$

$$\frac{d^2u}{dz^2} + z\frac{du}{dz} = 0$$

$v(x) = u(\sqrt{10}\,x)$