MATH 3363 EXAM 2. Sanders Spring 2008

This exam has 5 problems, and all 5 problems will be graded. Use my supplied paper only. Return your solution sheets with the problems in order. Put your name, **last name first**, and **student id number** on each solution sheet you turn in. Each problem is worth 20 points with parts equally weighted unless otherwise indicated.

1. Consider the following differential operators

(a) $\mathcal{L}(u) \equiv \dfrac{d^2u}{dx^2} + x\dfrac{du}{dx}$ (b) $\mathcal{L}(u) \equiv (x^2+1)\dfrac{d^2u}{dx^2} + x\dfrac{du}{dx}$

subject to boundary conditions $u(0) = u(1) = 0$. For both, determine the weight function $\omega(x)$ so that eigenfunctions associated to distinct eigenvalues are orthogonal with respect to the inner product $(u, v) = \int_0^1 u(x)v(x)\,\omega(x)\,dx$.

2. Solve **Laplace's equation** $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$ on the unit square $0 < x < 1,\ 0 < y < 1$ subject to the following boundary conditions.

(a) $u(x,0) = 0 \quad u(x,1) = 0$
$u(0,y) = 0 \quad u(1,y) = \sin(\pi y)$

(b) $u(x,0) = 0 \quad u(x,1) = \sin(5\pi x)$
$u(0,y) = 0 \quad u(1,y) = 0$

(c) $u(x,0) = 0 \quad u(x,1) = \sin(5\pi x)$
$u(0,y) = 0 \quad u(1,y) = \sin(\pi y)$
(6 points)

3. Solve the **heat equation** $\dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2}$ on the unit interval $0 < x < 1$ subject to the following boundary and initial conditions.

(a) $u(0,t) = 0 \quad u(1,t) = 1$
$u(x,0) = x + 2\sin(2\pi x)$

(b) $u(0,t) = 1 \quad u_x(1,t) = 1$
$u(x,0) = 1 + x + \sin(\frac{3}{2}\pi x)$

4. Solve the **wave equation** $\dfrac{\partial^2 u}{\partial t^2} = \dfrac{\partial^2 u}{\partial x^2}$ on the unit interval $0 < x < 1$ subject to following boundary and initial conditions.

(a) $u(0,t) = 1 \quad u(1,t) = 1$
$u(x,0) = 1$
$u_t(x,0) = \sin(\pi x)$

(b) $u(0,t) = 0 \quad u_x(1,t) = 1$
$u(x,0) = x + \sin(\frac{3}{2}\pi x)$
$u_t(x,0) = 0$

5. Solve the **2d heat equation** $\dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2}$ on the unit square $0 < x < 1,\ 0 < y < 1$ subject to the following boundary and initial conditions.

(a) $u(x,0,t) = 0 \quad u(x,1,t) = 0$
$u(0,y,t) = 0 \quad u(1,y,t) = 0$
$u(x,y,0) = \sin(2\pi x)\sin(5\pi y)$

(b) $u(x,0,t) = 0 \quad u(x,1,t) = 0$
$u_x(0,y,t) = 0 \quad u_x(1,y,t) = 0$
$u(x,y,0) = \sin(\pi y) + \cos(\pi x)\sin(2\pi y)$